Gyrokinetic Simulations of Tearing Instability

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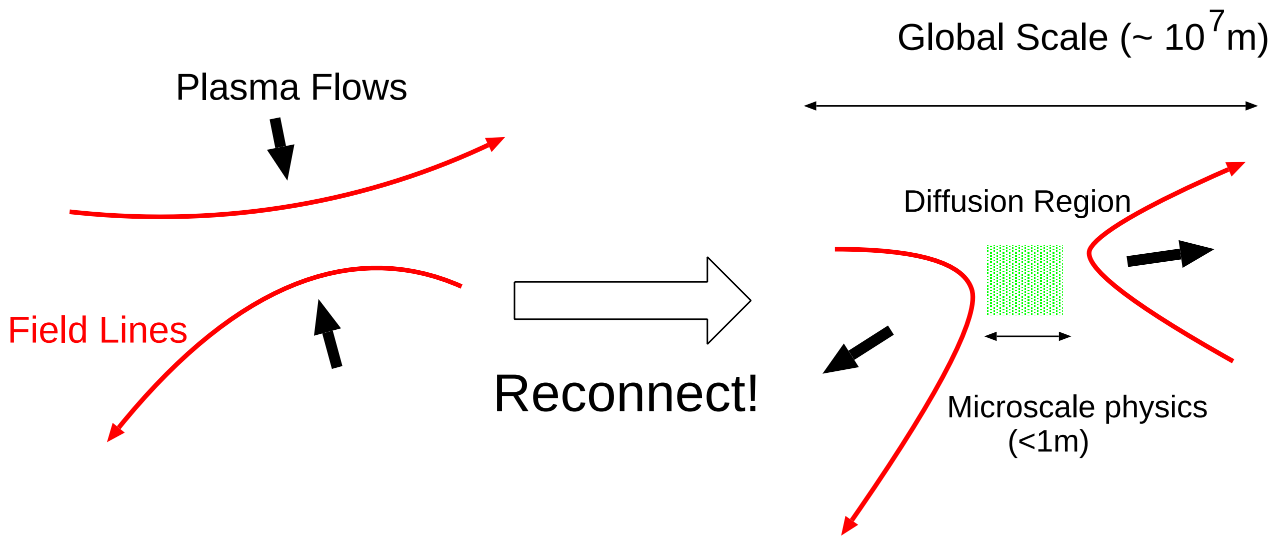
Outline

- Magnetic Reconnection and Tearing Instability
- Basic Theory for Tearing Instability
- Gyrokinetics
  - Collisions and Resistivity
  - `AstroGK` gyrokinetics code
- Numerical Simulations
  - Electron Physics
  - Ion Physics
  - Kinetic effect (ion temperature)
  - MHD limit
- Summary

Magnetic Reconnection

Magnetic reconnection is ubiquitous in plasmas

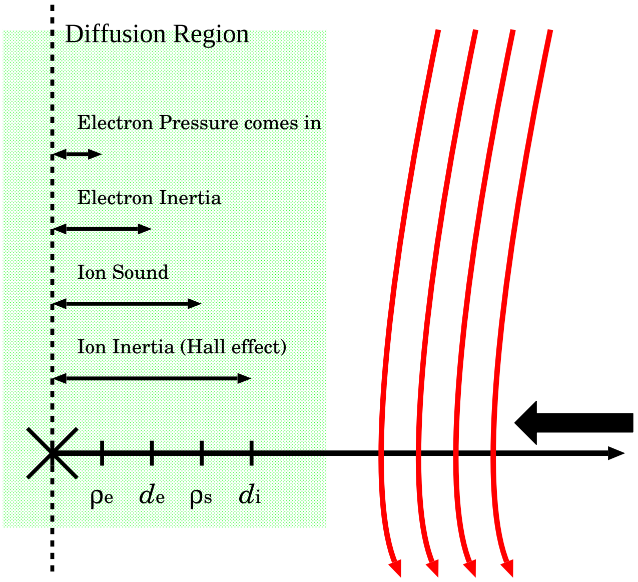


- Magnetic reconnection converts magnetic field energy into high speed plasma flows, heating of plasmas, and energetic particles.
- Sawtooth oscillations, island growth due to tearing mode, disruptions in fusion experimental devices (Degradation of confinement).
- Magnetospheric substorms, solar flares in astrophysical situation.

Magnetic Reconnection

Magnetic reconnection is a classic example where multiple physics (scales) are involved

- Physics to break flux-freezing is necessary for field lines to change its topology: primarily by collisions (resistivity).
- In collisionless environments, time scale of reconnection based on resistivity is far too slow to explain realistic explosive phenomena.
- Resistive spatial scale falls below kinetic scales (MHD theory is not valid).
- Global structure drastically changes depending on microscopic processes.



Questions

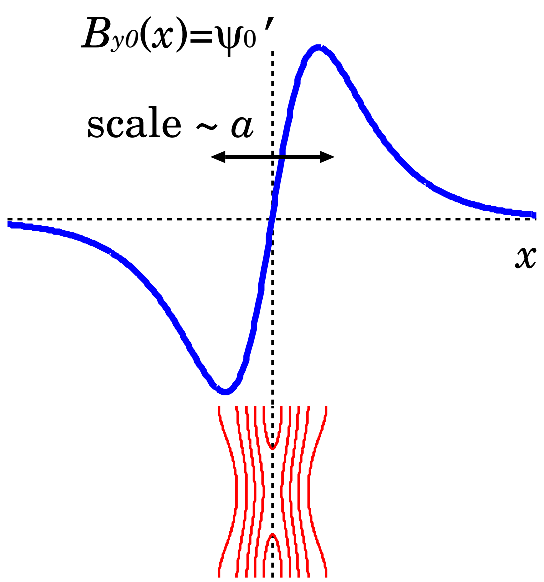
- What determines time scale of reconnection?
- What provides mechanism for field lines to reconnect?

Tearing Instability

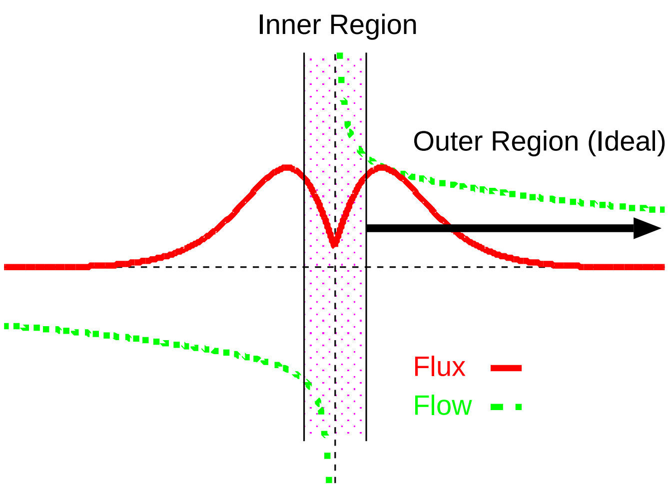
- Tearing instability is a resistive instability of current sheet configuration
- Spontaneous onset of reconnection process
- Linear stage
- Standard boundary layer or singular perturbation problem

Without resistivity and inertia, solution is singular

<u>Background current profile</u>



$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} - \left(k_y^2 + \frac{\psi_0'}{\psi_0'''}\right)\psi = 0 \qquad (1)$$



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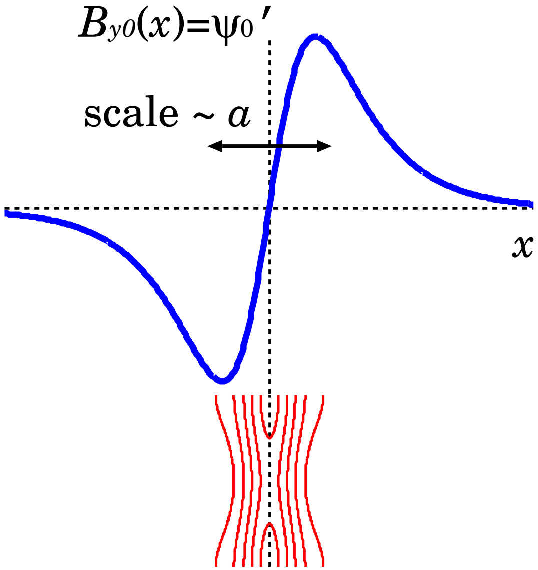

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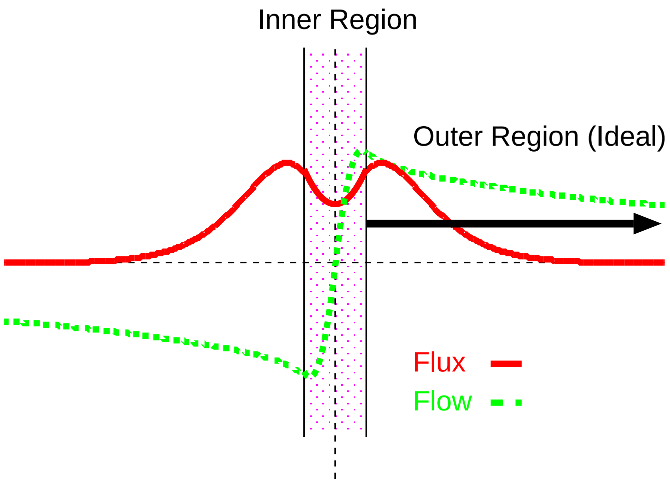

Tearing Instab. Theory (resistive MHD) [a]

Time Scales (normalized by $\tau_{\mathrm{A}} = a/V_{\mathrm{A}}$)

- Alfvén time (parallel to the reconnecting field): $\tau_{\mathrm{H}}/\tau_{\mathrm{A}} \equiv 1/ka$
- Resistive time scale: $\tau_{\mathrm{R}}/\tau_{\mathrm{A}} \equiv \mu_0 a V_{\mathrm{A}}/\eta \equiv S$: Lundquist number

Assumptions

- Time scale separations

$$1/\tau_{\mathrm{R}} \ll \gamma \ll 1/\tau_{\mathrm{H}} \tag{2}$$

- Scale separations

$$\ell \ll a \tag{3}$$

where ℓ is the resistive current layer width

Dispersion relation (matching two solutions)

$$\Delta' a = -\frac{\pi}{8}\gamma^{5/4}\tau_{\mathrm{H}}^{1/2}\tau_{\mathrm{R}}^{3/4}\frac{\Gamma((\lambda^{3/2}-1)/4)}{\Gamma((\lambda^{3/2}+5)/4)} \tag{4}$$

$\lambda = \gamma\tau_{\mathrm{H}}^{2/3}\tau_{\mathrm{R}}^{1/3}$, Δ' is a jump of ψ' providing instability criterion ($\Delta' > 0$).

[a] Furth, Killeen, and Rosenbluth, Phys. Fluids **6**, 459 (1963)

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Tearing Instab. Theory (More Physics)

- Hall effect (Ion inertia) [a] $\sim d_{\rm i}$ – Alfvén wave dispersion (Whistler)
- Pressure effect [b] (Ion Sound) $\sim \rho_s$ – Alfvén wave couples to sound wave
- Finite Larmor Radius (FLR) [c] $\sim \rho_{\rm i(e)}$ – Alfvén wave dispersion (KAW)
- Tensorial pressure $\sim \rho_{\rm e(i)}$ – break flux-freezing
- Electron inertia [d] $\sim d_{\rm e}$ – break flux-freezing

Some of the effects can exist in fluid models, while some are essentially kinetic.

Gyrokinetics

- No ad-hoc inclusion of detailed physics
  $\longrightarrow$ validation of various fluid models
- Collision physics – Do not assume resistivity
- Disadvantages: heavy, $\cdots$

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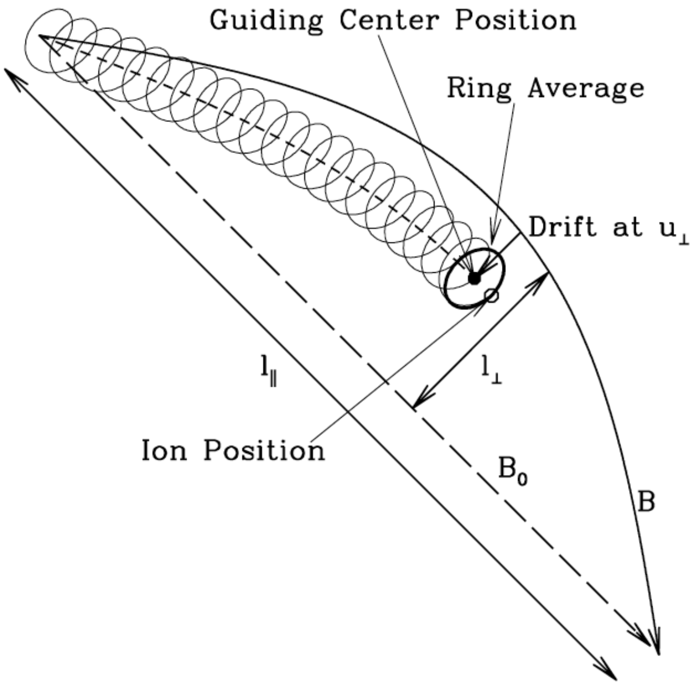
[a] e.g. Terasawa, GRL (1983), Fitzpatrick & Porcelli, PoP (2004)

[b] Coppi *et al.*, Nuclear Fusion (1966).

[c] Porcelli, PRL (1991)

[d] Schep *et al.* (1994)

Gyrokinetics



- Reduced kinetic model – 5 dimensional phase space
- Strong guide field (B_0) allows to separate out fast cyclotron motion
- Already multiscale
- Note: two-dimensional magnetic reconnection or tearing instability dynamics primarily independent of the guide field

Gyrokinetics: Basic equations

The distribution function of particles is given by $f = \left(1 - \frac{q\phi}{T_0}\right) f_0 + h$, where $f_0 = n_0/(\sqrt{\pi} v_{\rm th})^3 \exp(-v^2/v_{\rm th}^2)$ is the Maxwellian, and the thermal velocity is given by $v_{\rm th} = \sqrt{2T_0/m}$. The equations to solve are the gyrokinetic equation for $h = h(\boldsymbol{R}, V_\perp, V_\|)$,

$$\frac{\partial h}{\partial t} + V_\| \frac{\partial h}{\partial Z} + \frac{1}{B_0} \{\langle \chi \rangle_{\boldsymbol{R}}, h\} - \langle C(h) \rangle_{\boldsymbol{R}} = q \frac{f_0}{T_0} \frac{\partial \langle \chi \rangle_{\boldsymbol{R}}}{\partial t}, \tag{5}$$

$\chi = \phi - \boldsymbol{v} \cdot \boldsymbol{A}$ and the field equations for $\phi(\boldsymbol{r})$, $A_\|(\boldsymbol{r})$, and $\delta B_\|(\boldsymbol{r})$,

$$n_{\rm i} = n_{\rm e} \quad \Leftrightarrow \quad \sum_s \left[-\frac{q_s^2 n_{0s} \phi}{T_{0s}} + q_s \int \langle h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v} \right] = 0, \tag{6}$$

$$(\nabla \times \boldsymbol{B})|_\| = \mu_0 j_\| \quad \Leftrightarrow \quad \nabla_\perp^2 A_\| = -\mu_0 \sum_s q_s \int \langle h_s \rangle_{\boldsymbol{r}} v_\| \mathrm{d}\boldsymbol{v} \tag{7}$$

$$\nabla \cdot \left(\mathtt{P} + \mathtt{I} \frac{B_0 \delta B_\|}{\mu_0} \right) = 0 \quad \Leftrightarrow \quad B_0 \nabla_\perp \delta B_\| = -\mu_0 \nabla_\perp \cdot \sum_s \int \langle m \boldsymbol{v}_\perp \boldsymbol{v}_\perp h_s \rangle_{\boldsymbol{r}} \mathrm{d}\boldsymbol{v}. \tag{8}$$

Collision Operator

Recently, linearized collision operators for gyrokinetic simulations, which satisfies physical requirements are established and implemented in `AstroGK` [a].

The operators are the pitch-angle scattering (Lorentz), the energy diffusion, and moments conserving corrections to those operators for like-particle collisions. Electron-ion collisions consists of pitch angle scattering by background ions and ion drag are also included.

We, here, mainly discuss the electron-ion collisions since it contributes to resistivity. The operator is given by (in Fourier space)

$$C_{\mathrm{ei}}(h_{\mathrm{e},\boldsymbol{k}}) = \nu_{\mathrm{ei}} \left(\frac{v_{\mathrm{th,e}}}{V}\right)^3 \left(\frac{1}{2}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial h_{\mathrm{e},\boldsymbol{k}}}{\partial \xi} - \frac{1}{4}(1+\xi^2)\frac{V^2}{v_{\mathrm{th,e}}^2} k_\perp^2 \rho_{\mathrm{e}}^2 h_{\mathrm{e},\boldsymbol{k}} + \frac{2V_\parallel J_0(\alpha_{\mathrm{e}}) u_{\parallel,\mathrm{i},\boldsymbol{k}}}{v_{\mathrm{th,e}}^2} f_{0\mathrm{e}} \right) \tag{9}$$

We examine how this collision operator relates with resistivity which decays the current.

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[a] Abel *et al*, Phys. Plasmas **15**, 122509 (2008), arXiv:0808.1300; Barnes *et al*, accepted Phys. Plasmas (2009), arXiv:0809.3945v2.

Spitzer Resistivity

From the fluid picture current decays due to collisional resistivity as

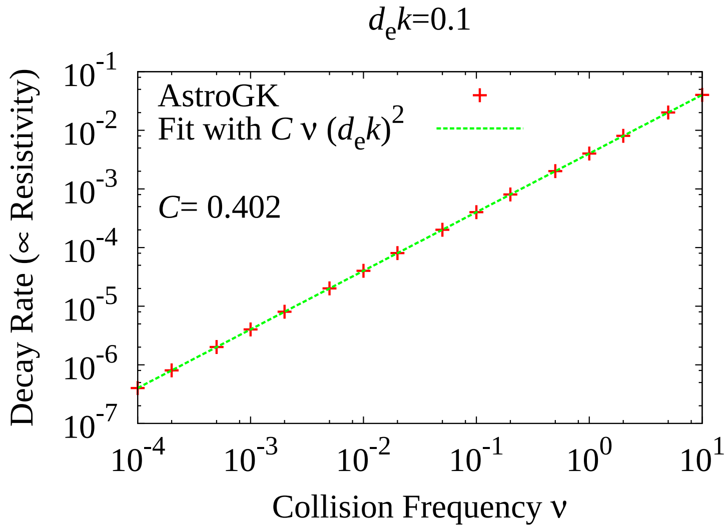
$$\frac{\partial j}{\partial t} = -\frac{\eta}{\mu_0} k^2 j, \qquad (10)$$

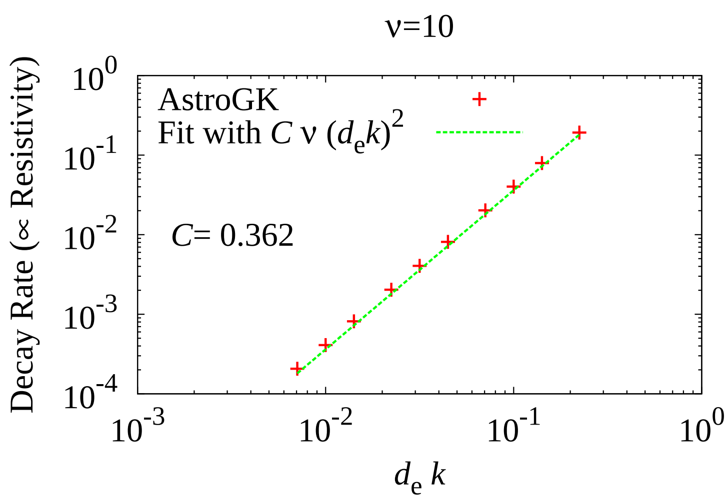
and the decay rate is $\tau_{\mathrm{decay}}^{-1} = (\eta/\mu_0)k^2$. Using the Spitzer resistivity given by $\eta = m_{\mathrm{e}}/(1.98\tau_{\mathrm{e}} n_{\mathrm{e}} e^2)$ where $\tau_{\mathrm{e}} = 3\sqrt{\pi}/(4\nu_{\mathrm{ei}})$, the decay rate is casted into the following form,

$$\tau_{\mathrm{decay}}^{-1} = C\nu_{\mathrm{ei}}(d_{\mathrm{e}}k)^2 \qquad (11)$$

where the constant $C = 4/(1.98 \times 3\sqrt{\pi}) \approx 0.380$.

Figures show dependence of decay rate on ν and $d_{\mathrm{e}}k$. Numerical estimated proportionality constant agrees with Spitzer's value within 5% error.

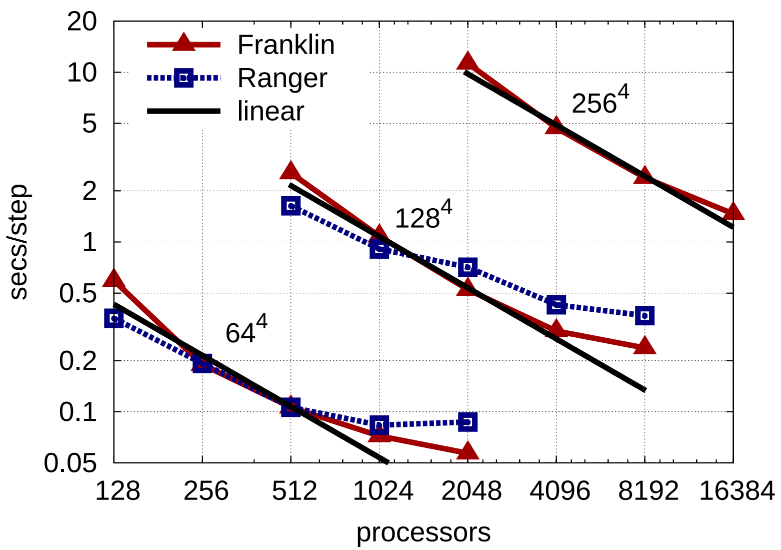




AstroGK

Publicly available at http://www.physics.uiowa.edu/˜ghowes/astrogk/ (paper in preparation)

- Eulerian (not particle)
- Fourier spectral in coordinate space (x,y)
- Periodic boundary in x and y
- Gaussian quadrature for velocity space integral
- Time integral:
  Implicit Euler for collisions
  Adams-Bashforth (3rd) for nonlinear terms
- Parallelized using MPI library
- NERSC (Franklin), NCSA (Jaguar), TACC (Ranger), etc



Simulation Setting

Parameters

$$r \equiv \lambda_{\rm i}/a, \qquad \sigma \equiv m_{\rm e}/m_{\rm i}, \tag{12}$$

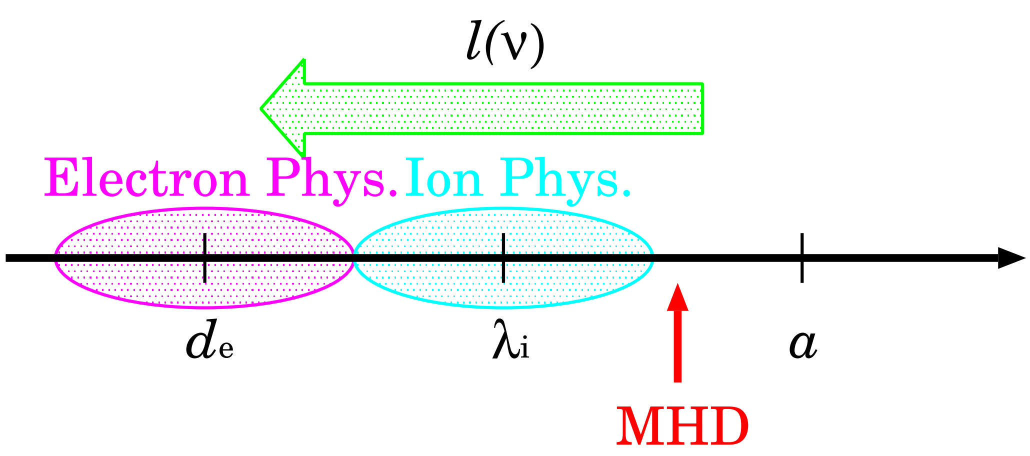
$$\tau \equiv T_{0\rm i}/T_{0\rm e}, \qquad \beta_{\rm e} \equiv n_0 T_{0\rm e}/(B_{\rm g}^2/2\mu_0) \tag{13}$$

$\lambda_{\rm i}$ is typical ion scale, $B_{\rm g}$ is the guide magnetic field

$$\rho_{\rm i}/\lambda_{\rm i} = \tau^{1/2}, \qquad d_{\rm i}/\lambda_{\rm i} = \beta_{\rm e}^{-1/2}, \qquad \rho_{\rm s}/\lambda_{\rm i} = \left(\frac{\Gamma}{2}(1+\tau)\right)^{1/2}, \tag{14}$$

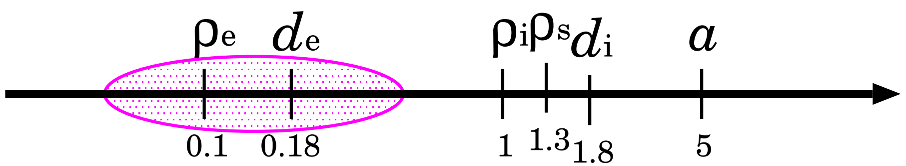
$$\rho_{\rm e}/\lambda_{\rm i} = \sigma^{1/2}, \qquad d_{\rm e}/\lambda_{\rm i} = \beta_{\rm e}^{-1/2}\sigma^{1/2}, \tag{15}$$

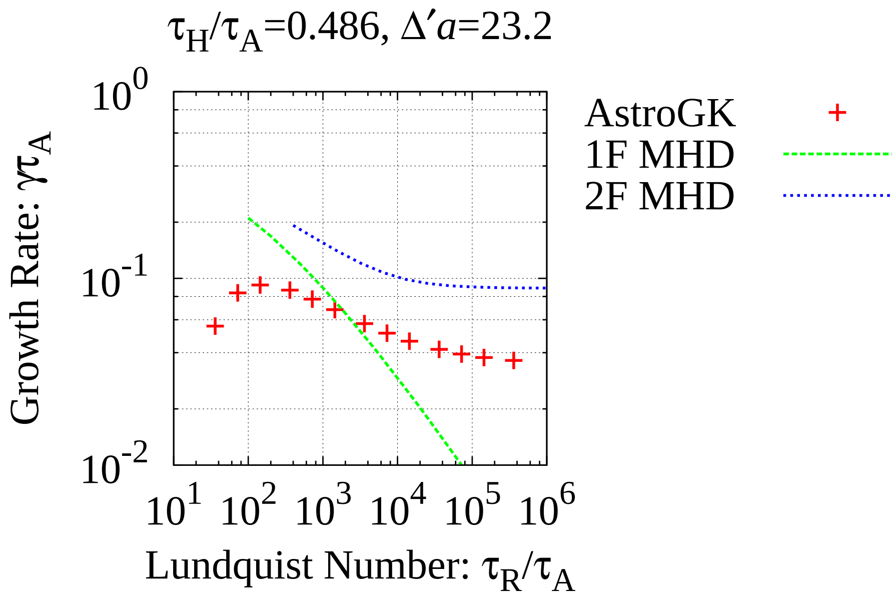
- Change collisionality ν to vary current layer width ℓ

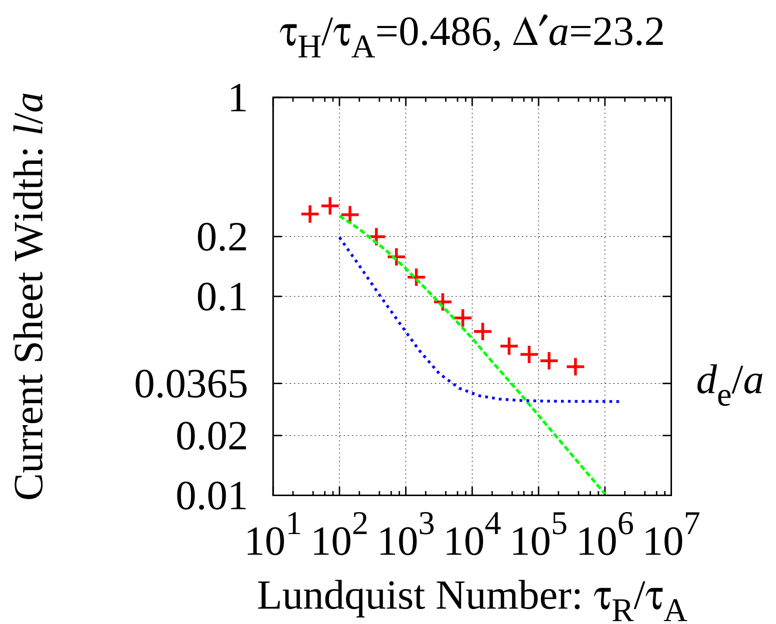


Electron Physics

$r = 0.2, \quad \sigma = 0.01, \quad \tau = 1, \quad \beta_e = 0.3$



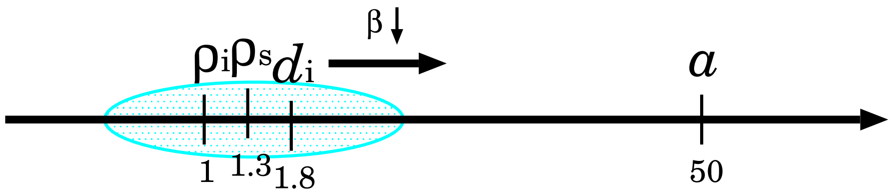


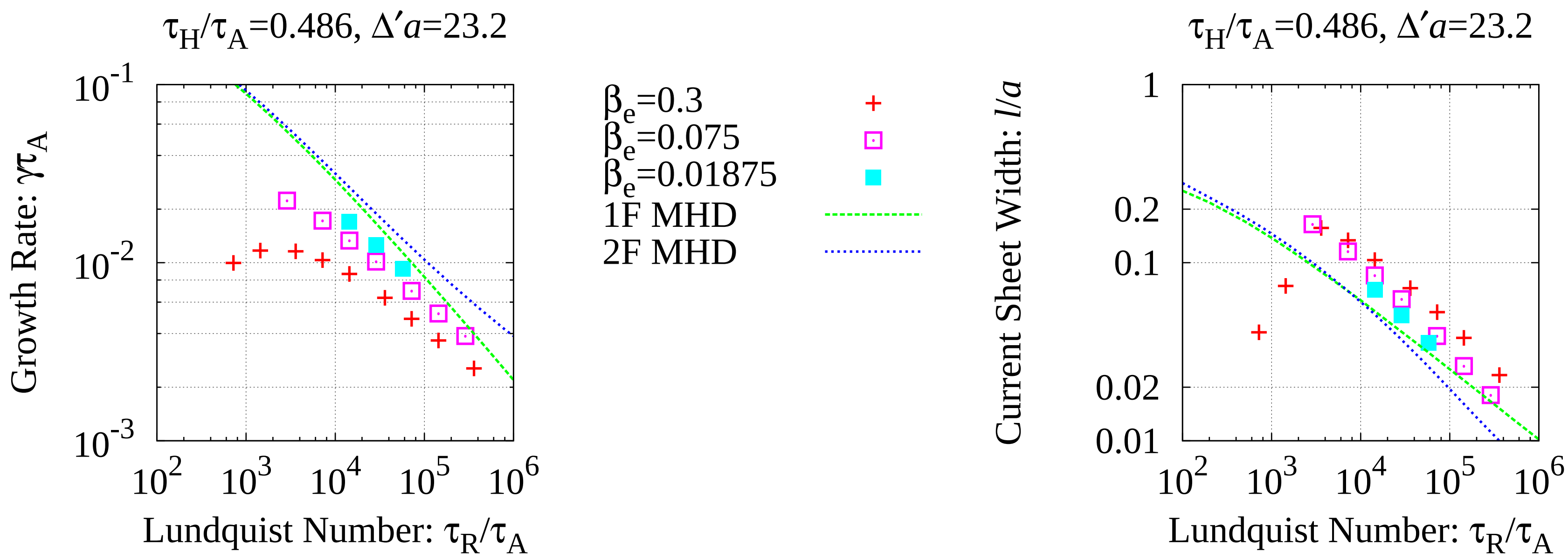


- Transition to collisionless reconnection – independent of collisions
- Electron inertia mediated – d_e sets lower bound of scale
- Difference between GK & 2F MHD may be ascribed to the treatment of pressure ($\Gamma \neq 5/3$)

Ion Physics

$r = 0.02, \quad \sigma = 0.01, \quad \tau = 1, \quad \beta_e = 0.3$

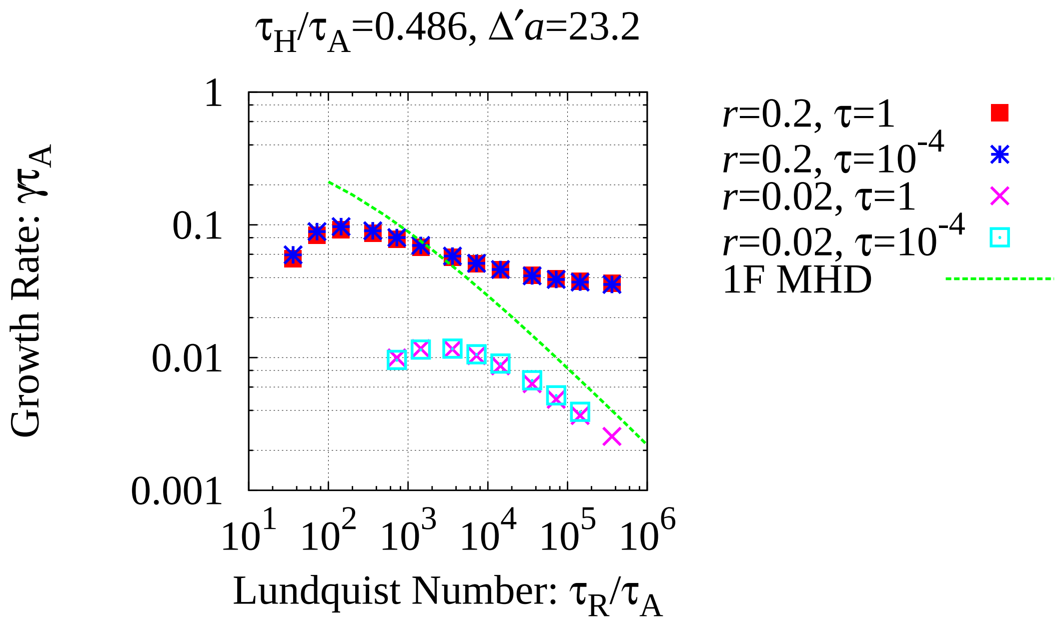




- Not MHD even if ion scale $\ll a$
- β_e slows down tearing growth
- Approaches to MHD limit as β decreases

Ion temperature

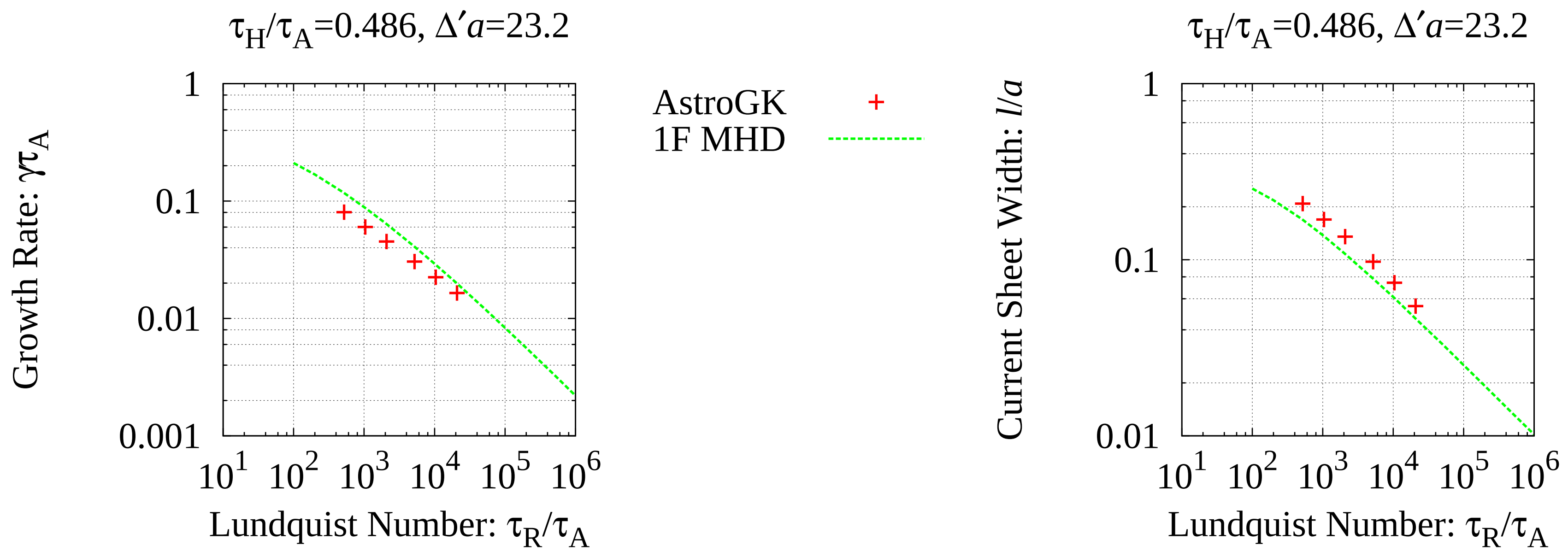
Cold ion ($\tau = 10^{-4}$) for different r ($r = 0.2$: electron phys., $r = 0.02$: ion phys.)
$\rightarrow \rho_{\rm i}$ becomes irrelevant ($< d_{\rm e}$)



- Ion temperature has no effect ($\rho_{\rm i} \lesssim \rho_{\rm s}$)

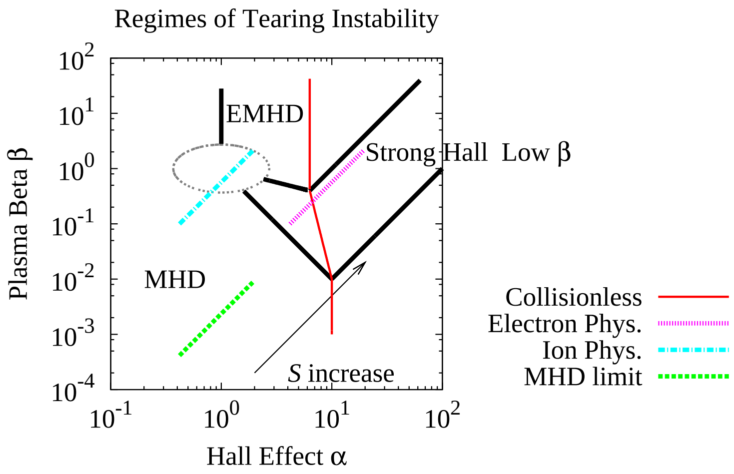
MHD limit

Low beta $\beta_e = 10^{-3}$, Cold ion $\tau = 10^{-4}$ case



Summary

- Magnetic reconnection and tearing instability are very good example of multiscale physics.
- We have performed collisionless and collisional tearing instability simulations, and have scanned for ν_{ei}.
- We have observed transition from collisional regime (collision dependent growth rate) to collisionless regime (collision independent growth rate).
- Equation of state need to be considered carefully – not simply $p \propto \rho^{\Gamma}$
- Understanding ion physics is rather difficult – pressure effect and Hall effect are tangled
- Ion temperature seems not playing significant role in the regimes considered
- GK simulation can also capture correct resistive MHD limit



Regimes of two fluid tearing instability obtained by Ahedo and Ramos [PPCF (2009)]