流れのあるプラズマの非線形シミュレーション – *double Beltrami* 平衡の形成

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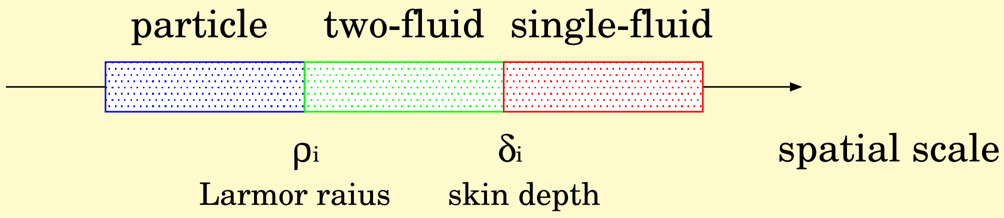
Introduction

Scale Hierarchy



- MHD
  Scale free
- Hall-MHD
  Intrinsic scale – ion collisionless skin depth ($\delta_\mathrm{i} \equiv c/\omega_\mathrm{pi}$)
  $\rightarrow$ chaos-induced resistivity [R.Numata and Z.Yoshida (2002)]

Plasma Flow

- MHD
  Alfvén singularity [E. Hameiri (1983), H. Tasso *et al.* (1998)]
- Hall-MHD
  Singular perturbation (Hall term) may avoid singularity

Double Beltrami State

Equilibrium in Hall-MHD is given under the Beltrami-Bernoulli conditions,

$$\boldsymbol{B} = a(\boldsymbol{V} - \epsilon\nabla \times \boldsymbol{B}), \quad \boldsymbol{B} + \epsilon\nabla \times \boldsymbol{V} = b\boldsymbol{V} \tag{1}$$

$$p + \frac{V^2}{2} = \text{const.} \tag{2}$$

where $\epsilon = \delta_{\mathrm{i}}/L$ ($\delta_{\mathrm{i}} \equiv c/\omega_{\mathrm{pi}}$: ion collisionless skin depth)

Relaxation Process [Z. Yoshida and S.M. Mahajan (2002)]

The relaxed state is achieved by minimizing a generalized enstrophy F (measure of fluctuations) with appropriate adjustment of ideal constants of motion $E,\ H_1,\ H_2$.

$$\delta(F - \mu_0 E - \mu_1 H_1 - \mu_2 H_2) = 0 \tag{3}$$

Adjustment condition to obtain the double Beltrami state $\quad E - \mu_1' H_1 - \mu_2' H_2 = 0$

$$\begin{aligned} F &= \tfrac{1}{2}\int |\nabla \times (\boldsymbol{A} + \epsilon\boldsymbol{V})|^2 \mathrm{d}x & E &= \tfrac{1}{2}\int (B^2 + V^2)\mathrm{d}x \\ H_1 &= \int \boldsymbol{A}\cdot\boldsymbol{B}\,\mathrm{d}x & H_2 &= \int (\boldsymbol{A} + \epsilon\boldsymbol{V})\cdot(\boldsymbol{B} + \epsilon\nabla \times \boldsymbol{V})\mathrm{d}x \end{aligned} \tag{4}$$

Simulation Model

We consider compressible Hall-MHD equations

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n\boldsymbol{V}) \tag{5}$$

$$\frac{\partial (n\boldsymbol{V})}{\partial t} = -\nabla \cdot (n\boldsymbol{V}\boldsymbol{V}) - \nabla p + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} \tag{6}$$

$$+\frac{1}{R_{\mathrm{e}}}\left[\nabla^2 \boldsymbol{V} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{V})\right] \tag{7}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[\boldsymbol{V} \times \boldsymbol{B} - \frac{\epsilon}{n}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}\right] + \frac{1}{R_{\mathrm{m}}}\nabla^2 \boldsymbol{B} \tag{8}$$

$$\frac{\partial p}{\partial t} = -(\boldsymbol{V} \cdot \nabla)p - \gamma p \nabla \cdot \boldsymbol{V} \tag{9}$$

$$+\frac{1}{R_{\mathrm{e}}}|\nabla \times \boldsymbol{V}|^2 + \frac{4}{3R_{\mathrm{e}}}|\nabla \cdot \boldsymbol{V}|^2 + \frac{1}{R_{\mathrm{m}}}|\nabla \times \boldsymbol{B}|^2 \tag{10}$$

where n: number density (ion mass is normalized to be unity), p: ion pressure, R_{e}: Reynolds number, R_{m}: magnetic Reynolds number, γ: ratio of specific heat. We have assumed the quasi-neutrality $n = n_{\mathrm{i}} = n_{\mathrm{e}}$, and $p_e = 0$.

Simulation Model

All simulations are carried out by NEC SX7 installed in TCSC, NIFS (30GFLOPS,7MByte)

Simulation Domain

3D rectangular domain $[-a, a] \times [-a, a] \times [0, 2\pi R]$

Boundary Condition

Periodic in z direction

Rigid perfect conducting wall in x, y direction

$$\boldsymbol{V} = 0 \tag{11}$$

$$\boldsymbol{B}_{\mathrm{N}} = 0$$

$$\partial_N \boldsymbol{B}_{\mathrm{T}} = 0 \;\; (\boldsymbol{n} \times (\nabla \times \boldsymbol{B}) = 0) \tag{12}$$

where $\boldsymbol{n}$ is a unit normal vector, ∂_N is a normal derivative, and $B_{\mathrm{N}}, B_{\mathrm{T}}$ is the normal and tangential component of the magnetic field, respectively.

To assure tangential components of the electric field vanish, we set $\epsilon = 0$ at the wall.

Numerical Method

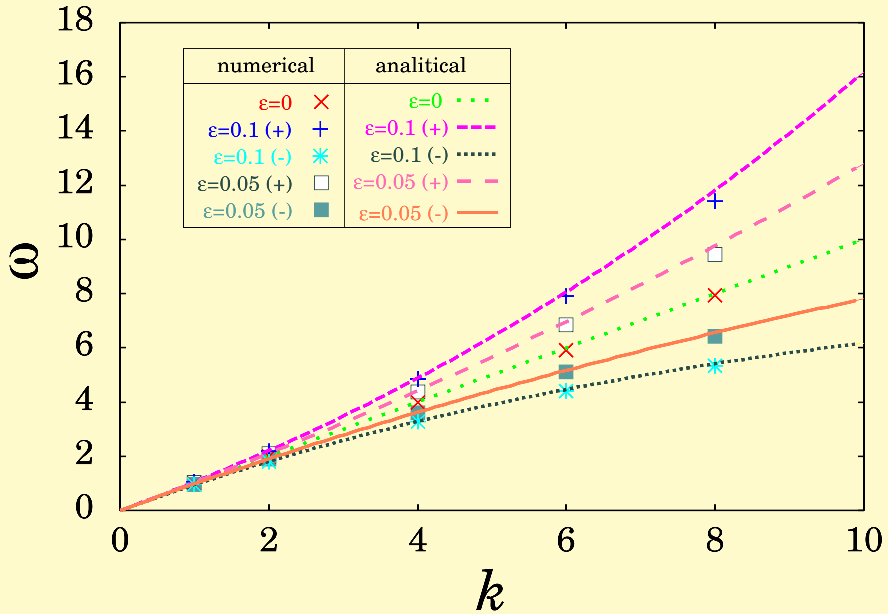
Finite difference (2nd order) + Runge-Kutta-Gill method

Dispersion Relation

In the ideal limit, the dispersion relation for purely transversal small perturbation around a uniform equilibrium ($\boldsymbol{B}_0 = (0, 0, B_0), \boldsymbol{V}_0 = 0, \rho_0, p_o$ are constants) is given by

$$\omega = \pm \frac{V_{\mathrm{A}}}{2} \left[\epsilon k^2 \pm \sqrt{\epsilon^2 k^4 + 4k^2} \right] . \tag{13}$$

The shear Alfvén wave splits into the Alfvén whistler (right-hand polarized) wave and the Alfvén ion cyclotron (left-hand polarized) wave.



Initial Condition

2D force free equilibrium ($\nabla \times \boldsymbol{B} = k_0 \boldsymbol{B}$) [R.Horiuchi and T.Sato (1985)]

Magnetic Field

$$B_x = -\frac{1}{k_0}(k_1 B_1 \cos k_2 x \sin k_1 y + k_2 B_2 \cos k_1 x \sin k_2 y) \quad (14)$$

$$B_y = \frac{1}{k_0}(k_2 B_1 \sin k_2 x \cos k_1 y + k_1 B_2 \sin k_1 x \cos k_2 y) \quad (15)$$

$$B_z = B_1 \cos k_2 x \cos k_1 y + B_2 \cos k_1 x \cos k_2 y \quad (16)$$

Flow

$$\boldsymbol{V} = V_z \boldsymbol{e}_z = M_{\mathrm{A}} B_z \boldsymbol{e}_z \quad (17)$$

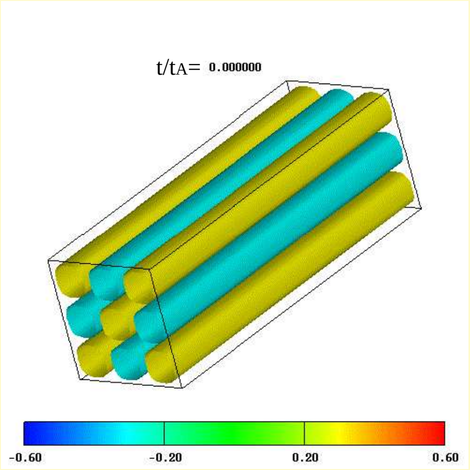
Density

$$\rho = 1 \text{ (uniform)} \quad (18)$$

Pressure

$$p = p_0 \text{ (uniform)} \quad (19)$$

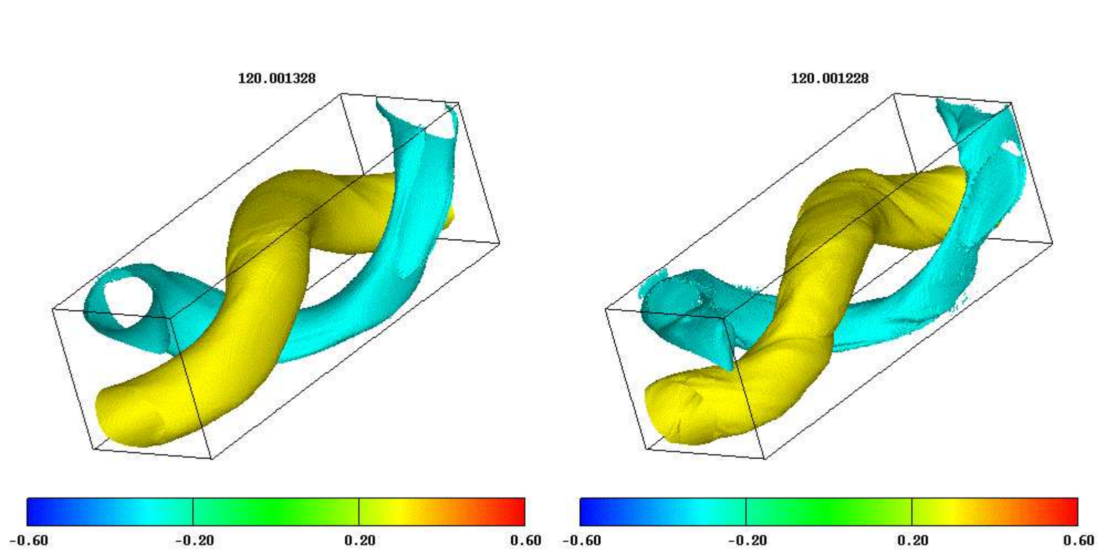
$$\beta = \int p_0 \mathrm{d}x / \int \frac{1}{2} B^2 \mathrm{d}x \quad (20)$$



Bz isosurface

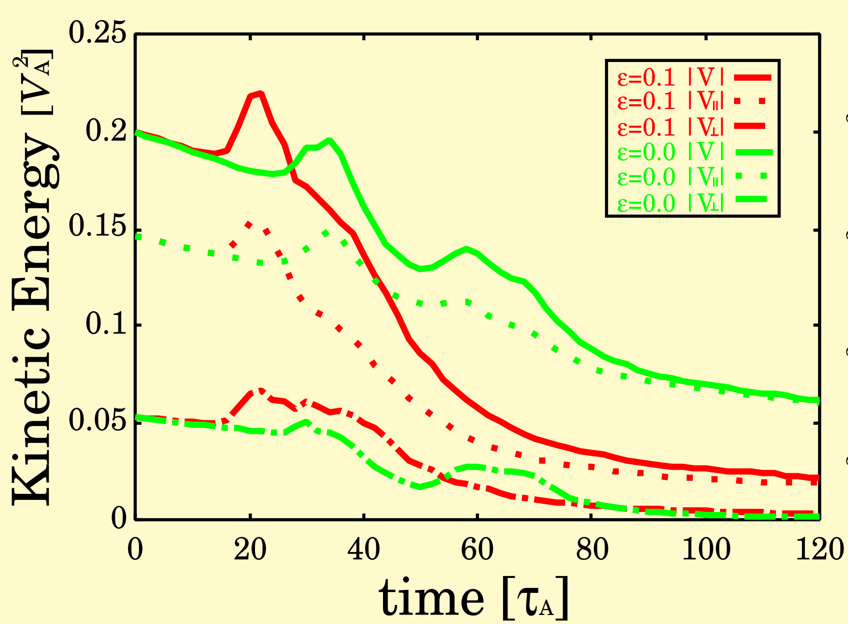
Equilibrium State

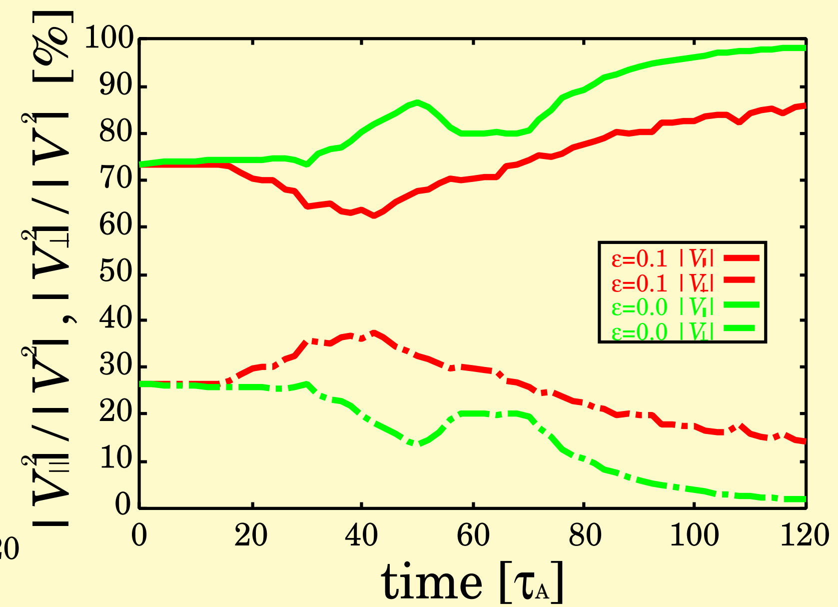
Aspect Ratio : $2\pi R/2a = 3$
Grid : $129 \times 129 \times 256$
$R_{\rm e} = 10^4, R_{\rm m} = 10^4$
$\beta = 3.0, M_{\rm A} = 0.5\ \epsilon = 0, 0.1$



left panel : $\epsilon = 0.1$ *(Hall-MHD), right panel :* $\epsilon = 0$ *(MHD)*

Kinetic Energy



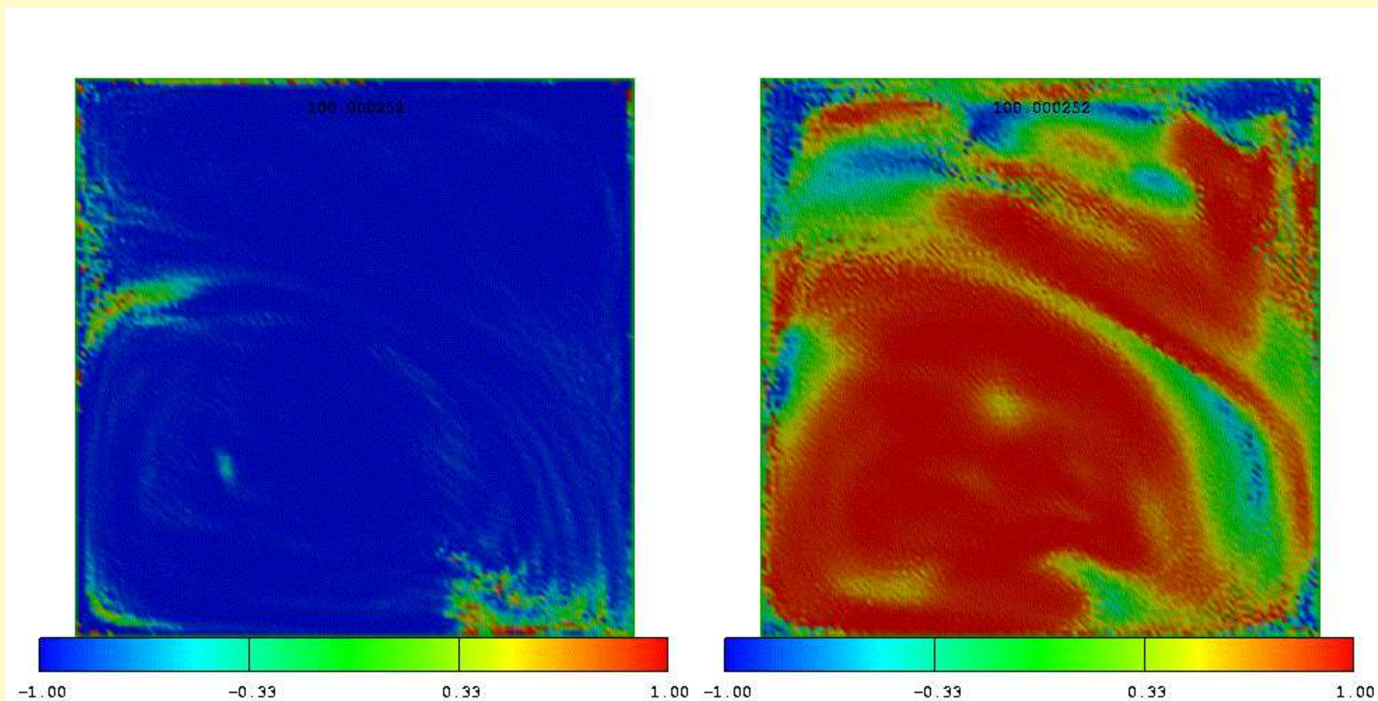


- $\epsilon = 0.1$ (Hall-MHD): $V_\perp$ is more than 10% of $|V|$
- $\epsilon = 0.0$ (MHD): No $V_\perp$ remains

Beltrami Conditions

$$\boldsymbol{B} = a(\boldsymbol{V} - \frac{\epsilon}{n}\nabla \times \boldsymbol{B}), \quad \boldsymbol{B} + \epsilon\nabla \times \boldsymbol{V} = b\boldsymbol{V} \tag{21}$$

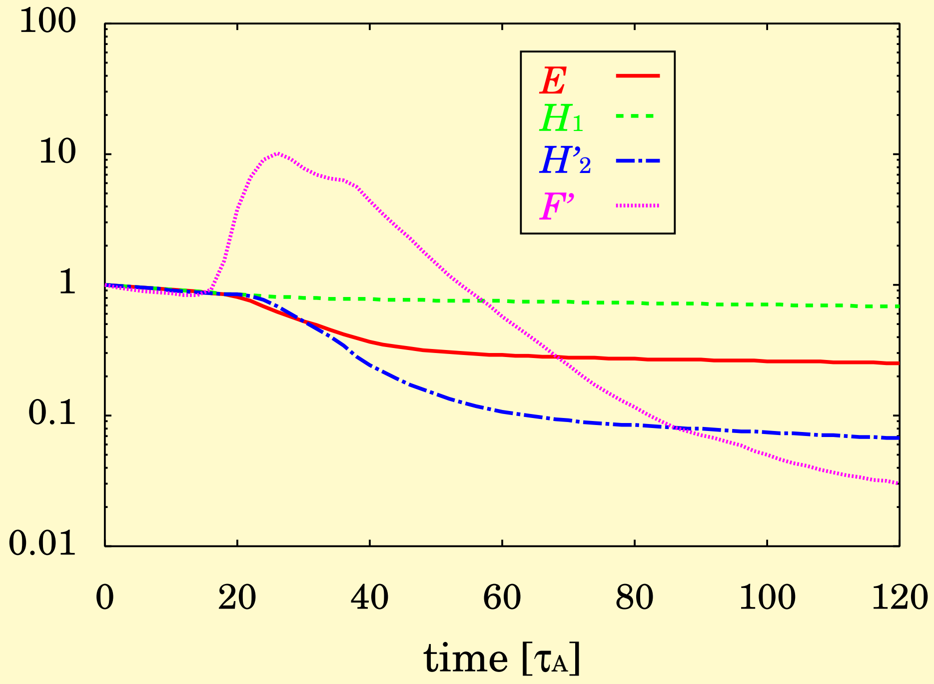
Time : $100.0\tau_{\mathrm{A}}$



(a) distribution of a (b) distribution of b

Variational Principle

Temporal evolution of the macroscopic quantities $F' \equiv \int |\nabla \times \boldsymbol{V}|^2 \mathrm{d}x$, E, H_1, and $H_2' \equiv H_2 - H_1$ normalized by the initial value for the case of $\epsilon = 0.1$.



The coercivity condition demands that the order of fragility is $H_1 > E > H_2 - H_1$, which agree with the numerical result.

Summary

- We have developed a nonlinear 3D Hall-MHD simulation code.
- Dispersion relation in the Hall-MHD is reproduced by the simulation code.
- Comparing the two-fluid relaxed state with single-fluid one, an appreciable flow including perpendicular component was created.
- Relaxation process is investigated by means of the variational principle. Energy is not a minimizer.

	with Hall term	without Hall term
ideal constants of motion	Ion Helicity $H_2' = \int (2\epsilon \boldsymbol{V} \cdot \boldsymbol{B} + \epsilon^2 \boldsymbol{V} \cdot \nabla \times \boldsymbol{V}) \mathrm{d}x$ $\Downarrow$	Cross Helicity $H_c = \int \boldsymbol{V} \cdot \boldsymbol{B} \mathrm{d}x$ $\Downarrow$
Relaxed State	Double Beltrami	Single Beltrami + Parallel Flow