

Onset of Turbulence in a Drift Wave-Zonal Flow System

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Introduction

Numerical analyses of bifurcation phenomena in the Hasegawa-Wakatani model [1] are presented, that provide new insights into the interactions between turbulence and zonal flows in tokamak edge region

Bifurcations in the model correspond to the onset of drift wave driven turbulence, generation of zonal flows, and re-emergence of drift wave turbulence as the zonal flows become unstable

These phenomena are resistive drift wave turbulence analog of the Dimits shift [2] in ion-temperature-gradient driven turbulence

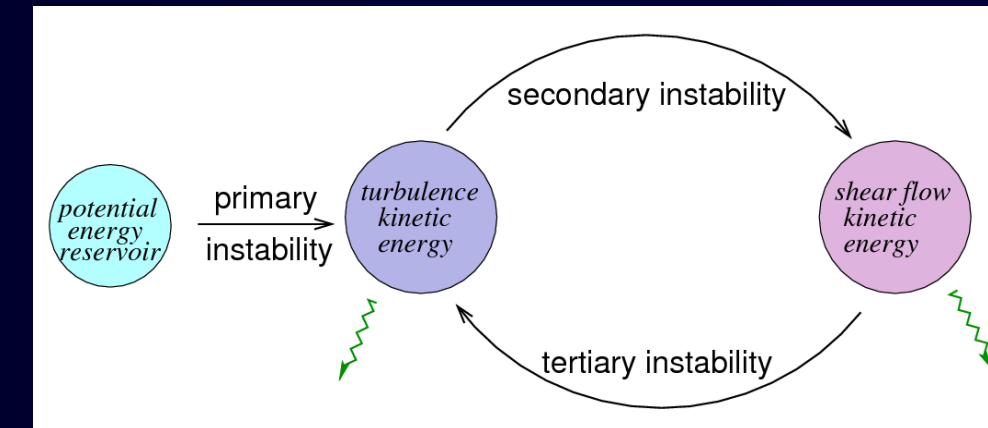


Fig. 1: Schematic diagram of interactions among energetic subsystems

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Modified Hasegawa-Wakatani Model

HW model describes evolution of density fluctuation (n) and vorticity ($\zeta = \nabla^2 \varphi$) in tokamak edge region, and includes effects of nonuniform background density ($\kappa \equiv \partial \ln n_0(x) / \partial x$) and parallel electron motion ($\alpha \equiv -T_e J(\eta n_0 \omega_{ci} e^2) \partial^2 / \partial z^2$)

$$\frac{\partial \zeta}{\partial t} + [\varphi, \zeta] = \alpha(\varphi - n) - D \nabla^4 \zeta$$

$$\frac{\partial n}{\partial t} + [\varphi, n] = \alpha(\varphi - n) - \kappa \frac{\partial \varphi}{\partial y} - D \nabla^4 n$$

Coupling term comes from parallel Ohm's law

$$j_y = -en v_{e,z} = -\frac{1}{\eta} \frac{\partial}{\partial z} \left(\varphi - \frac{T_e}{e} \ln n \right)$$

In 2D setting α becomes constant by the replacement $\partial / \partial z \rightarrow i k_z$. Since zonal components ($k_y = k_z = 0$) do not contribute to the parallel current, it must be removed from the coupling term [3]

$$\alpha(\varphi - n) \rightarrow \alpha(\tilde{\varphi} - \tilde{n})$$

$$\text{Zonal: } \langle f \rangle = \frac{1}{L_y} \int f dy, \text{ Nonzonal: } \tilde{f} = f - \langle f \rangle$$

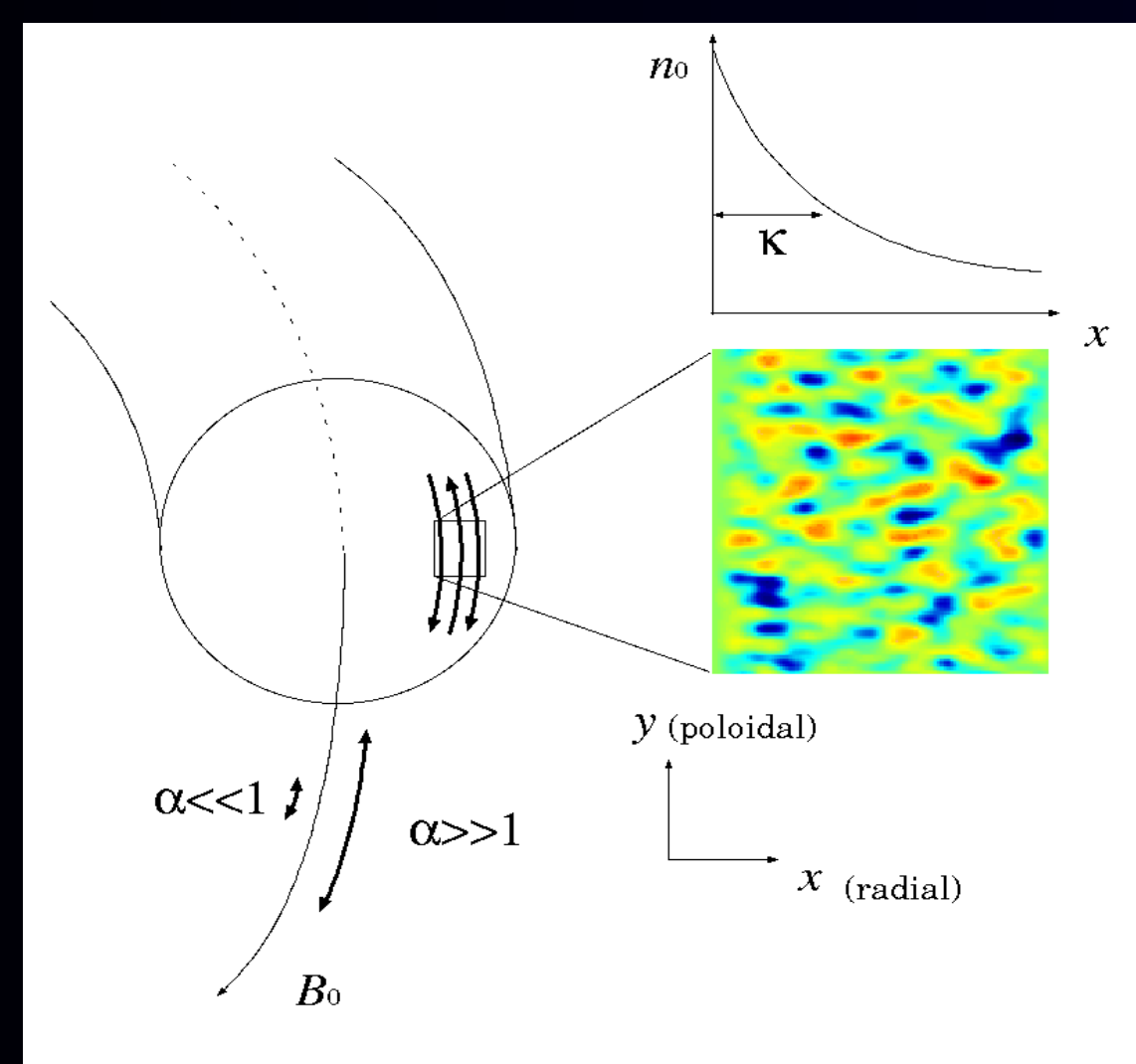


Fig. 2: Schematic view of setting of HW model

Simulation Results

MHW equations are solved numerically in a doubly periodic square slab domain

i) Modification is crucial for zonal flow generation, and resulting turbulence suppression [4]

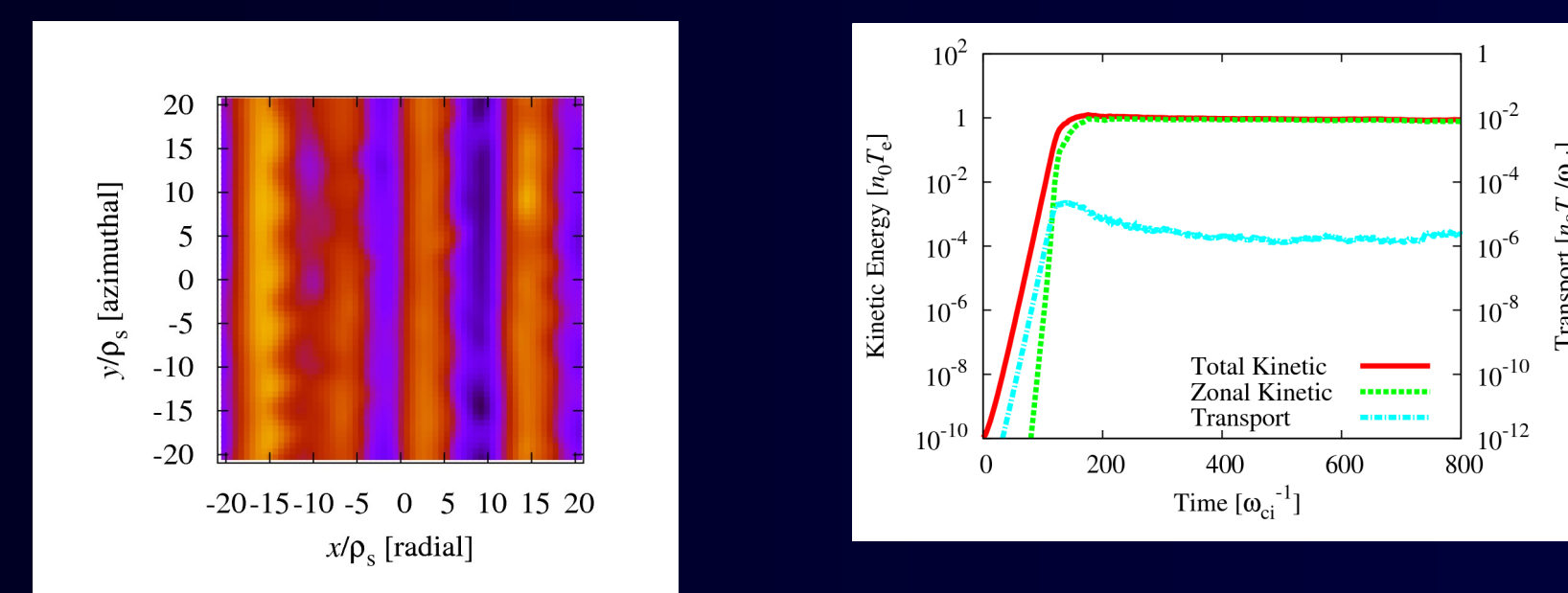


Fig. 3: Contour plot of saturated electrostatic potential (left), and time evolutions of total kinetic energy, zonal flow kinetic energy, and transport (right)

ii) Conversion from zonal-flow-dominated state to turbulence-dominated state occurs over a narrow range of parameter space

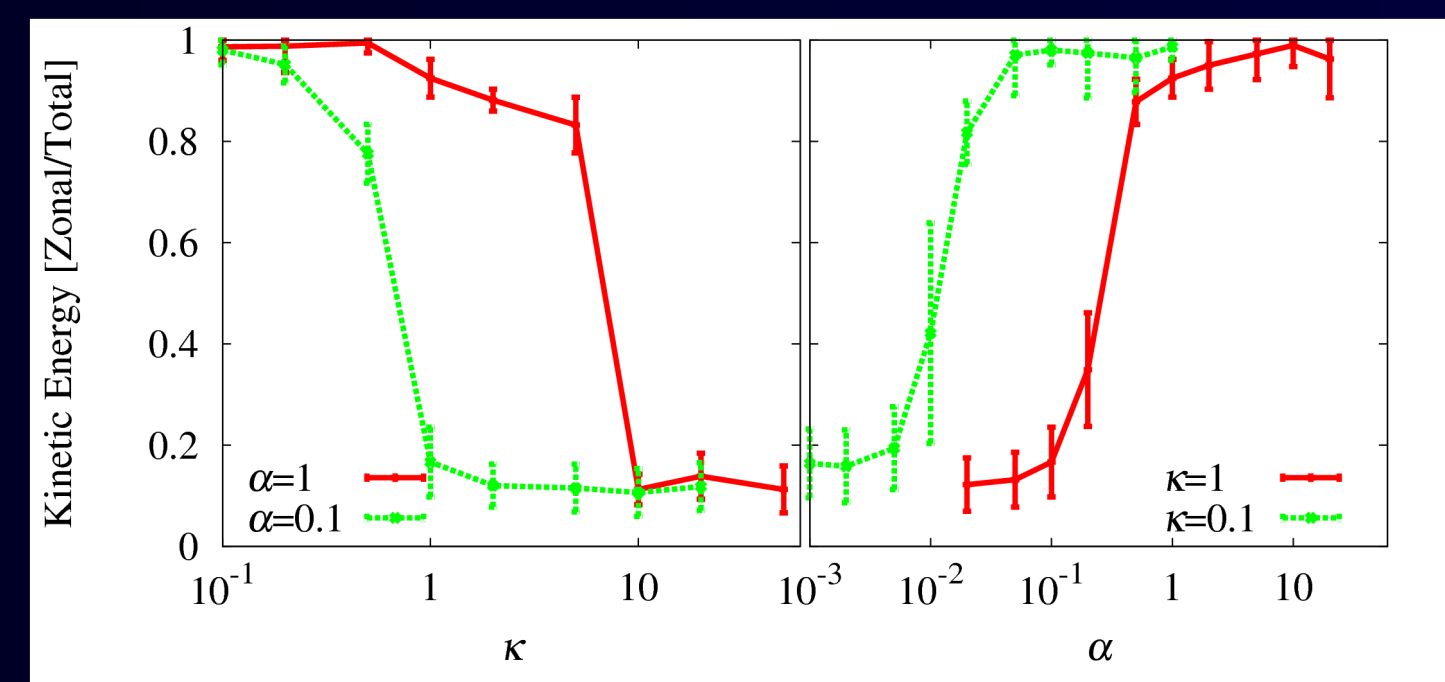


Fig. 4: Parameter dependence of the zonal flow kinetic energy normalized by the total kinetic energy

iii) Generated flow profile is determined for stability analysis we assume

$$V(x) = V_0 \sin(\lambda x)$$

Amplitude and flow shear are estimated from numerical results

$$V_0 \propto \kappa^2, \text{ and } \lambda \approx 0.3$$

Stability of Zonal Flows

Kelvin-Helmholtz stability of generated zonal flow is examined and compared with numerically obtained transition points

Variables are decomposed as $\varphi = \varphi_0(x) + \tilde{\varphi}(x) e^{i(k_y y - \omega t)}$, and $n = \tilde{n}(x) e^{i(k_y y - \omega t)}$ where $d\varphi_0/dx = V$ gives background flow in y direction. Linearization of MHW eqns. yields an eigenvalue eqn.

$$\left[\frac{d^2}{dx^2} - k_y^2 + \frac{k_y V''}{\omega - k_y V} - \frac{i\alpha}{\omega - k_y V + i\alpha} \left(1 - \frac{k_y \kappa}{\omega - k_y V} \right) \right] \tilde{\varphi} = 0$$

i) In adiabatic ($\alpha \rightarrow \infty$) and hydrodynamic ($\alpha \rightarrow 0$) limits, some conditions for stability are known (see [5] and references therein)

ii) α stabilizes zonal flow. Positive eigenvalue disappears at $\alpha \approx 0.000417$

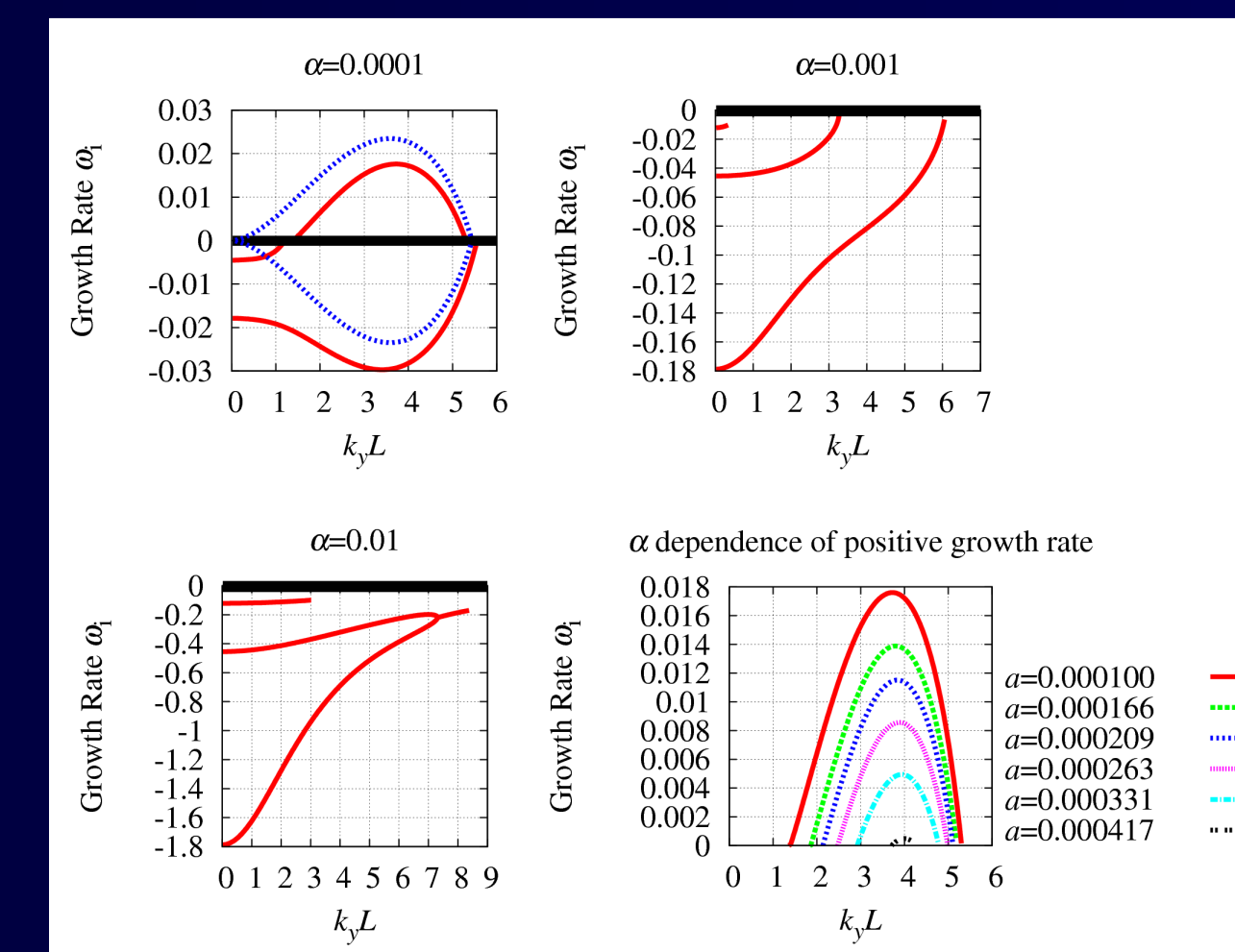
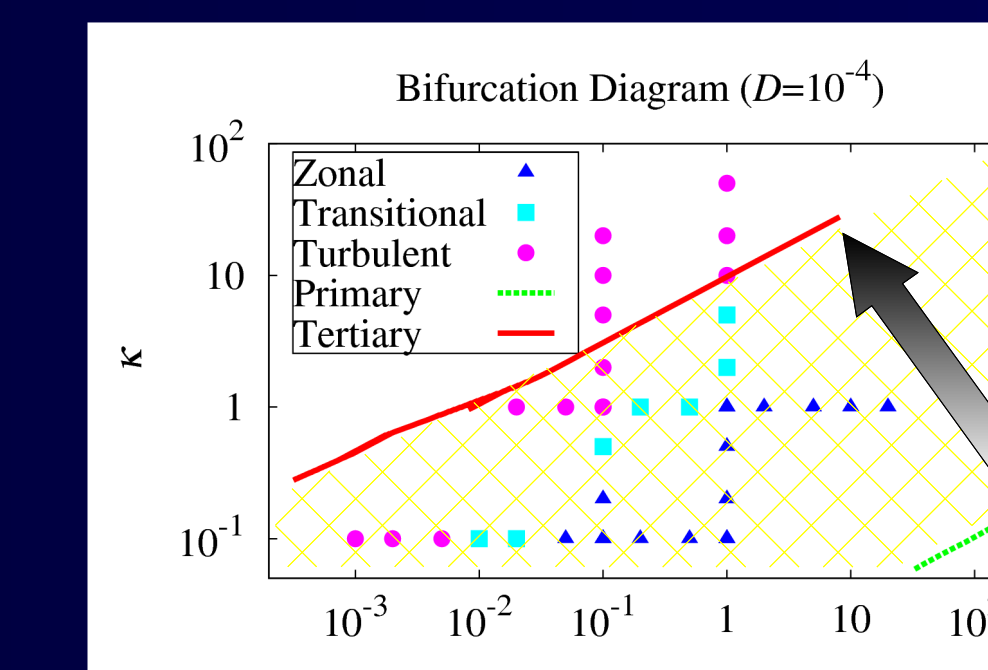


Fig. 5: Imaginary part of eigenvalues as function of k_y for finite α

iii) κ effect is minor because it appears as $\kappa \alpha$, except that it controls flow amplitude

iv) Bifurcation diagram shows the correlation between the linearized stability estimates and the regimes observed in numerical simulations



Upshift of turbulence onset due to zonal flow

Fig. 6: Bifurcation diagram in α - κ space

Summary & Conclusions

We have analyzed bifurcation phenomena in two-dimensional resistive drift wave turbulence using modified Hasegawa-Wakatani model.

We have shown that, in the MHW model, zonal flows are self-organized and suppress turbulence and turbulent transport over a range of parameters beyond the linear stability threshold for resistive drift waves.

By performing a systematic parameter survey, we have found that zonal-flow-dominated states suddenly disappear as a threshold is crossed, being replaced by a turbulence-dominated state.

The threshold of the onset of turbulence has been compared with the linear stability threshold of an assumed laminar zonal flow profile.

Numerical analysis of the eigenvalue problem determining the stability of the assumed zonal flow profile have revealed that i) κ determines the amplitude of the zonal flows, thus, large κ destabilizes the zonal flow, ii) the adiabatic response of parallel electrons given by α stabilizes the zonal flows.

Constructed bifurcation diagram in the α - κ plane confirms the scenario of the onset of turbulence in the drift wave-zonal flow system being due to the disruption of zonal flows by Kelvin-Helmholtz instability.

This is the first observation of an upshift of turbulence onset in the resistive drift wave system, which is analogous to the well-known Dimits shift in ion-temperature-gradient driven turbulence.

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