Two-Fluid Nonlinear Simulation of Self-Organization of Plasmas with Flows

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Introduction

Flows in Plasmas

- Laboratory Plasmas, Fusion Plasmas
  (Non-neutral Plasmas, Tokamak H-mode boundary layer, Shear-flow Stabilization)
- Space Plasmas
  (Magnetic reconnection, Solar corona, High beta equilibrium of the Jupiter)

Single-Fluid Model

- Singularity in equilibrium equation

$$L = \int \left[\frac{1}{2}(1 - M^2(\psi))|\nabla\psi|^2 - F(\psi) \right] \mathrm{d}x \Rightarrow -\Delta[\psi - \phi(\psi)] = F'(\psi) \qquad (1)$$

  E. Hameiri, Phys. Fluids, **26**(1), 230 (1983)
  H. Tasso, *et. al.*, Phys. Plasmas **5**, 2378 (1998)

- Scale invariance
  Current sheet model to heating the solar corona
  E.N. Parker, Astrophys. J. **471**, 489 (1996)

Two-Fluid MHD

The Hall term leads a nonlinear singular perturbation, which

- puts anchor on unlimited scale conversions induced by nolinearity.
- avoids singularity.

$\Rightarrow$ cf. viscosity in neutral fluids

Double Beltrami Equilibrium

$$\frac{\partial}{\partial t}\boldsymbol{\omega}_j - \nabla \times (\boldsymbol{U}_j \times \boldsymbol{\omega}_j) = 0 \quad (j = 1, 2) \tag{2}$$

$$\begin{aligned} \boldsymbol{\omega}_1 &= \boldsymbol{B}, & \boldsymbol{\omega}_2 &= \boldsymbol{B} + \epsilon \nabla \times \boldsymbol{V} \\ \boldsymbol{U}_1 &= \boldsymbol{V} - \epsilon \nabla \times \boldsymbol{B}, & \boldsymbol{U}_2 &= \boldsymbol{V} \end{aligned} \tag{3}$$

$\epsilon = \delta_{\mathrm{i}}/L$ ($\delta_{\mathrm{i}} \equiv c/\omega_{\mathrm{pi}}$: ion collisionless skin depth)

Beltrami Bernoulli Condition

$$\boldsymbol{U}_j \times \boldsymbol{\omega}_j = 0 = \nabla \varphi_j \quad (j = 1, 2) \tag{4}$$

Double Betrami Equilibrium

The general solution is described by a linear combination of eigenfunctions of the curl operator $\boldsymbol{G}_\pm$ ($\nabla \times \boldsymbol{G}_\pm = \lambda_\pm \boldsymbol{G}_\pm$) :

$$\boldsymbol{B} = C_+\boldsymbol{G}_+ + C_-\boldsymbol{G}_-, \ \ \boldsymbol{V} = (a^{-1} + \epsilon\lambda +)C_+\boldsymbol{G}_+ + (a^{-1} + \epsilon\lambda -)C_-\boldsymbol{G}_- \tag{5}$$

The generalized Bernoulli condition translates as

$$p + \frac{V^2}{2} = \mathrm{const.} \tag{6}$$

Application

- H-mode boundary layer
  S.M. Mahajan and Z. Yoshida, Phy. Plasmas **7**, 635 (2000).
- Eruptive events in solar coronas
  S. Ohsaki, *et. al.*, Astrophys. J. **559**, L61 (2001).
- Chaos-induced dissipation in collisionless magnetic reconnection
  R. Numata and Z. Yoshida, Phys. Rev. Lett. **88**, 045003 (2003).

Relaxation Process

Global Invariants

$$\text{Energy} \qquad E = \frac{1}{2}\int (B^2 + V^2) dx \tag{7}$$

$$\text{Electron Helicity} \qquad H_1 = \frac{1}{2}\int \boldsymbol{A}\cdot\boldsymbol{B} dx \tag{8}$$

$$\text{Ion Helicity} \qquad H_2 = \frac{1}{2}\int (\boldsymbol{A} + \epsilon\boldsymbol{V})\cdot(\boldsymbol{B} + \epsilon\nabla\times\boldsymbol{V}) dx \tag{9}$$

Variational Principle

Z.Yoshida and S.M.Mahajan, Phys. Rev. Lett., **88**, 095001 (2002)

$$\delta(F - \mu_0 E - \mu_1 H_1 - \mu_2 H_2) = 0 \tag{10}$$

$$F = \frac{1}{2}\int |\nabla\times(\boldsymbol{A} + \epsilon\boldsymbol{V})|^2 dx \tag{11}$$

$$\delta(E - \mu_1' H_1 - \mu_2' H_2) = 0 \tag{12}$$

μ_0, μ_1, μ_2 are Lagrange multipliers. The relaxation process is realized by minimizing perturbation F with appropriate adjustment of $E,\ H_1,\ H_2$.

Simulation Model

We consider compressible Hall-MHD equations

$$
\begin{aligned}
\frac{\partial n}{\partial t} &= -\nabla \cdot (n\boldsymbol{V}) && (13)\\
\frac{\partial (n\boldsymbol{V})}{\partial t} &= -\nabla \cdot (n\boldsymbol{V}\boldsymbol{V}) - \nabla p + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} && (14)\\
&\quad + \frac{1}{R_{\mathrm{e}}}\left(\nabla^2 \boldsymbol{V} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{V})\right) && (15)\\
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times \left[\boldsymbol{V} \times \boldsymbol{B} - \frac{\epsilon}{n}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}\right] + \frac{1}{R_{\mathrm{m}}}\nabla^2 \boldsymbol{B} && (16)\\
\frac{\partial p}{\partial t} &= -(\boldsymbol{V} \cdot \nabla)p - \gamma p \nabla \cdot \boldsymbol{V} && (17)\\
&\quad + \frac{1}{R_{\mathrm{e}}}|\nabla \times \boldsymbol{V}|^2 + \frac{4}{3R_{\mathrm{e}}}(\nabla \cdot \boldsymbol{V})^2 + \frac{1}{R_{\mathrm{m}}}|\nabla \times \boldsymbol{B}|^2 && (18)
\end{aligned}
$$

where n: number density (ion mass is normalized to be unity), p: ion pressure, R_{e}: Reynolds number, R_{m}: magnetic Reynolds number, γ: ratio of specific heat. We have assumed the quasi-neutrality $n = n_{\mathrm{i}} = n_{\mathrm{e}}$, and $p_e = 0$.

Simulation Model

Simulation Domain

3 dimensional rectangular domain $[-a, a] \times [-a, a] \times [0, 2\pi R]$

Boundary Condition

Periodic in z direction

Rigid perfect conducting wall in x, y direction

$$\boldsymbol{v} = 0 \tag{19}$$

$$\boldsymbol{B}_{\mathrm{N}} = 0$$

$$\partial_N \boldsymbol{B}_{\mathrm{T}} = 0 \ \ (\boldsymbol{n} \times (\nabla \times \boldsymbol{B}) = 0) \tag{20}$$

where $\boldsymbol{n}$ is a unit normal vector, ∂_N is a normal derivative, and $B_{\mathrm{N}}, B_{\mathrm{T}}$ is the normal and tangential component of the magnetic field, respectively.

To assure tangential components of the electric field vanish, we set $\epsilon = 0$ at the wall.

Numerical Method

2nd order central difference in space, the Runge-Kutta-Gill method for time integration

2nd order smoothing to suppress short wave length mode

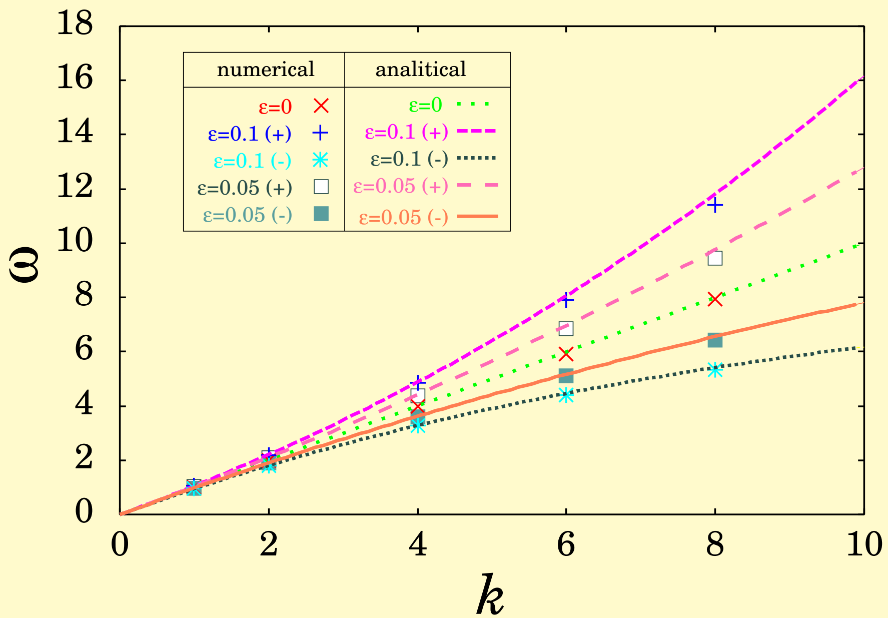
add a diffusion in the equation of continuity to keep density positive

Dispersion Relation

In an ideal limit, the dispersion relation for purely transversal small perturbation around a uniform equilibrium ($\boldsymbol{B}_0 = (0, 0, B_0), \boldsymbol{V}_0 = 0, \rho_0, p_o$ are constants) is given by

$$\omega = \frac{V_\mathrm{A}}{2}\left[\pm\epsilon k^2 \pm \sqrt{\epsilon^2 k^4 + 4k^2}\right], \tag{21}$$

which describes the Alfvén whistler wave. We have solved numerically this dispersion relation in a periodic domain in order to confirm a validity of the code.



Initial Condition

2D force free equilibrium ($\nabla \times \boldsymbol{B} = k_0 \boldsymbol{B}$)
R.Horiuchi and T.Sato, Phys. Rev. Lett., **55**, 211 (1985)

Magnetic Field

$$B_x = -\frac{1}{k_0}(k_1 B_1 \cos k_2 x \sin k_1 y + k_2 B_2 \cos k_1 x \sin k_2 y) \quad (22)$$

$$B_y = \frac{1}{k_0}(k_2 B_1 \sin k_2 x \cos k_1 y + k_1 B_2 \sin k_1 x \cos k_2 y) \quad (23)$$

$$B_z = B_1 \cos k_2 x \cos k_1 y + B_2 \cos k_1 x \cos k_2 y \quad (24)$$

Flow

$$\boldsymbol{V} = V_z \boldsymbol{e}_z = M_{\mathrm{A}} B_z \boldsymbol{e}_z \quad (25)$$

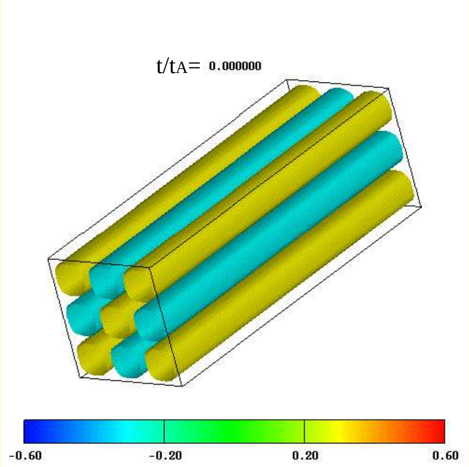
Density

$$\rho = 1 \ \text{(uniform)} \quad (26)$$

Pressure

$$p = p_0 \ \text{(uniform)} \quad (27)$$

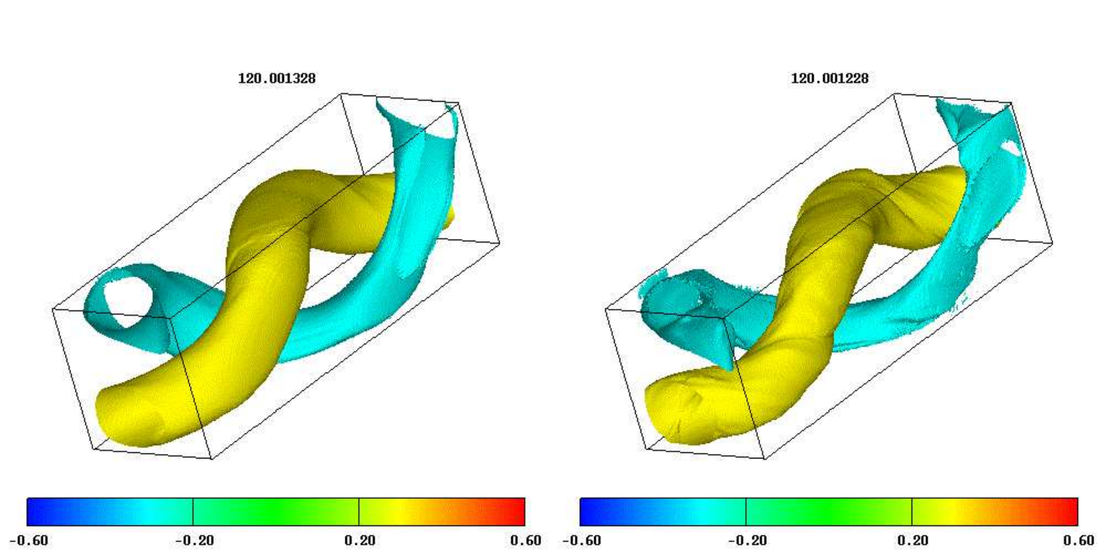
$$\beta = \int p_0 \mathrm{d}x \Big/ \int \frac{1}{2} B^2 \mathrm{d}x \quad (28)$$



Bz isosurface

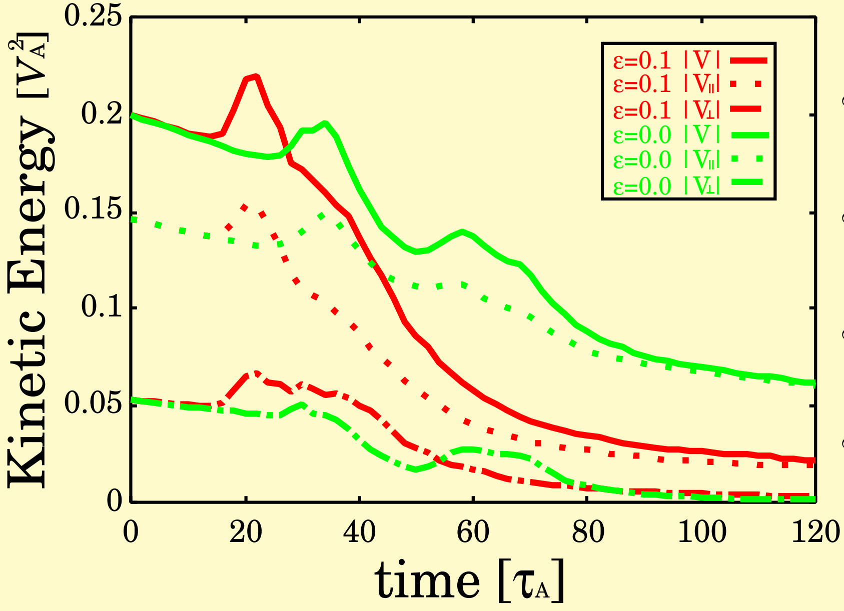
Equilibrium State

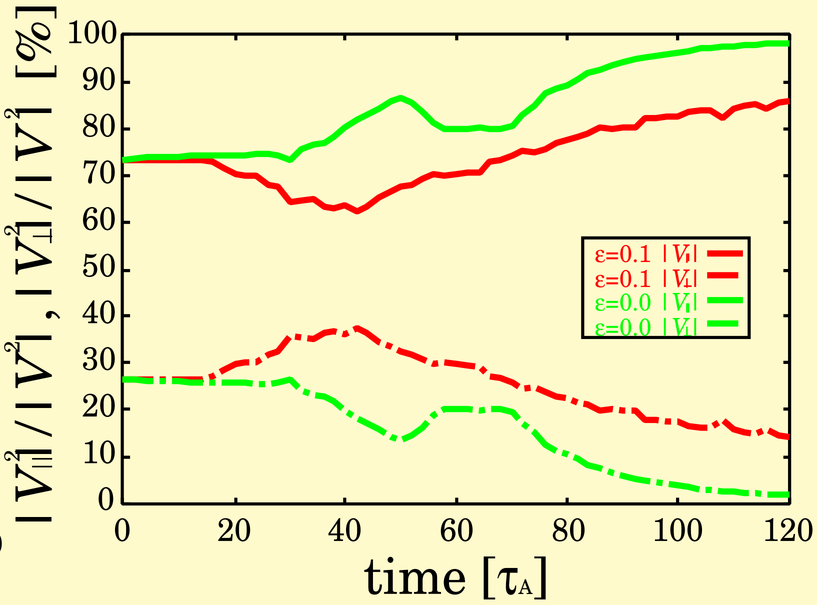
Aspect Ratio : $2\pi R/2a = 3$
Grid : $129 \times 129 \times 256$
$R_{\mathrm{e}} = 10^4, R_{\mathrm{m}} = 10^4$
$\beta = 3.0, M_{\mathrm{A}} = 0.5\ \epsilon = 0, 0.1$



left panel : $\epsilon = 0.1$ (Hall-MHD), right panel : $\epsilon = 0$ (MHD)

Kinetic Energy



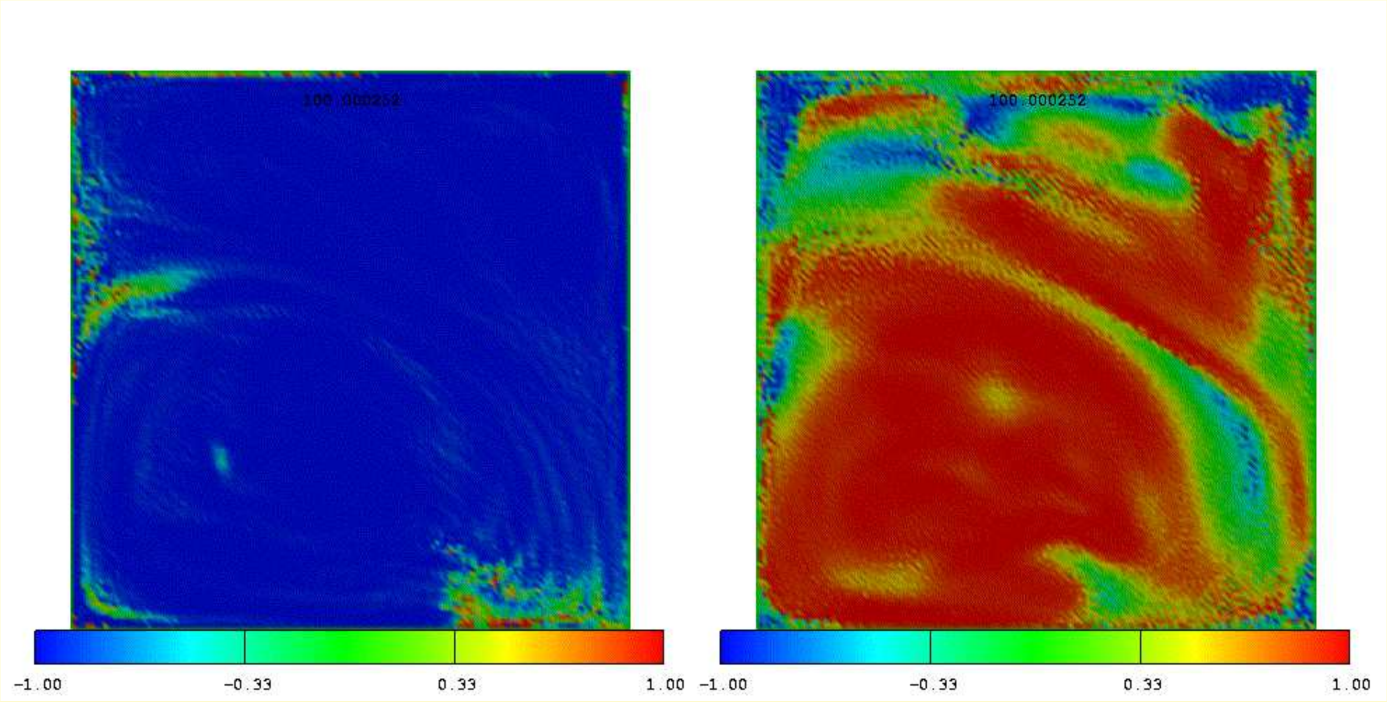


- $\epsilon = 0.1$ (Hall-MHD): $V_\perp$ is more than 10% of $|V|$
- $\epsilon = 0.0$ (MHD): No $V_\perp$ remains

Beltrami Conditions

$$\boldsymbol{B} = a(\boldsymbol{V} - \frac{\epsilon}{n}\nabla \times \boldsymbol{B}), \quad \boldsymbol{B} + \epsilon\nabla \times \boldsymbol{V} = b\boldsymbol{V} \tag{29}$$

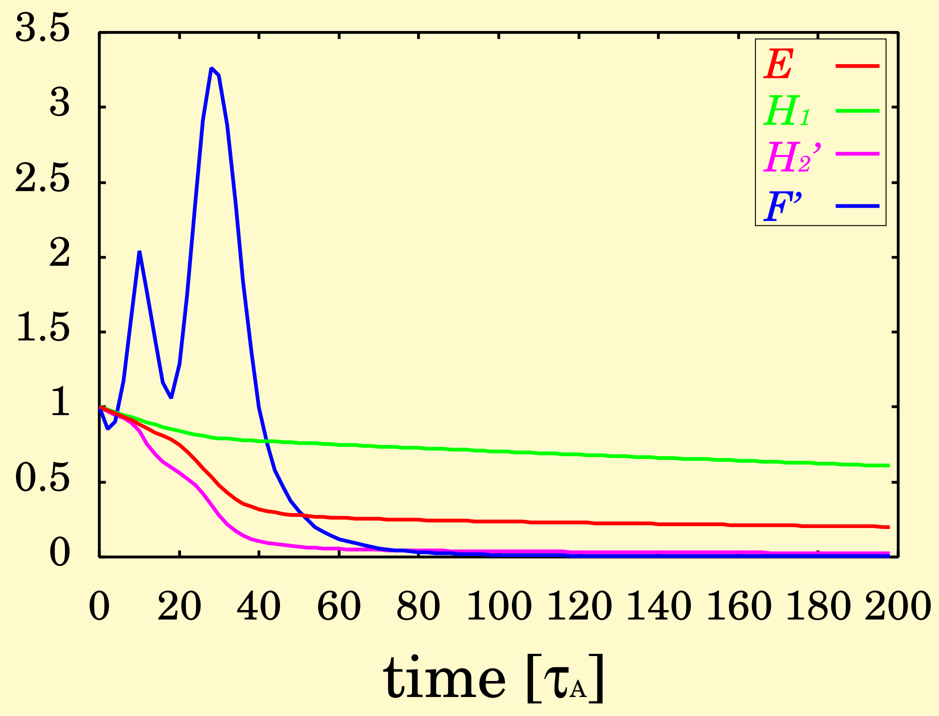
Time : $100.0\tau_{\mathrm{A}}$



(a) distribution of a (b) distribution of b

Variational Principle

Temporal evolution of the conservative quantities and the generalized enstrophy normalized by the initial value for the case of $\epsilon = 0.1$



The coercivity condition demands that the order of fragility is $H_1 > E > H_2 - H_1$, which agree with the numerical result.

Summary

- We have developed a nonlinear 3D Hall-MHD simulation code.
- The dispersion relation of the Alfvén whistler wave is reproduced.
- Comparing the two-fluid relaxed state with single-fluid one, an appreciable flow including perpendicular component was created.
- Relaxation process is investigated by means of the variational principle. Energy is not a minimizer.