Nonlinear Simulation of Drift Wave Turbulence

14th Gaseous Electronics Meeting, February 2006

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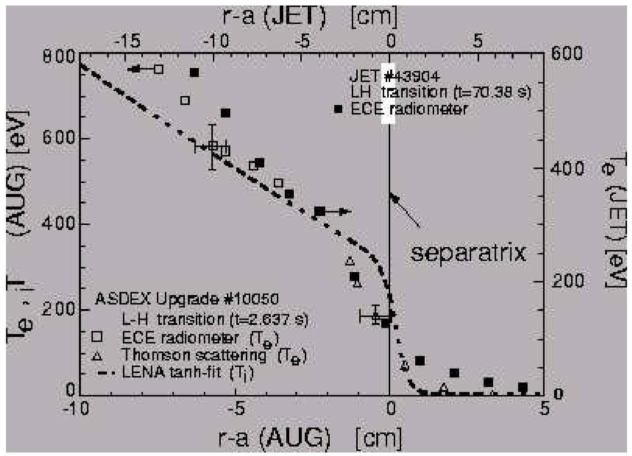
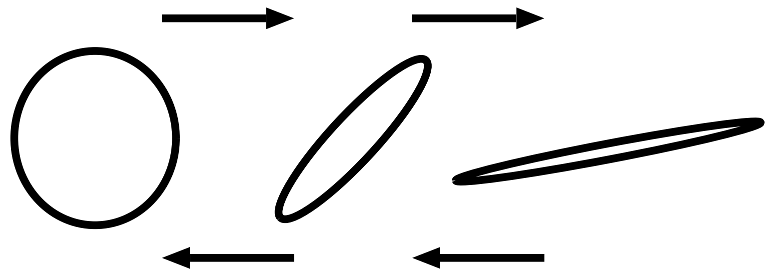
L-H Transition and Transport

- In magnetically confined plasma, spontaneous transition to a suppressed – transport regime often observed (called L-H transition)
- L-H transition is characterized by steep gradient of density/temperature at the plasma edge
- Rough estimate of transport $\sim \Delta x^2/\Delta t$
- Shear flow reduces size of convective cell which determines transport



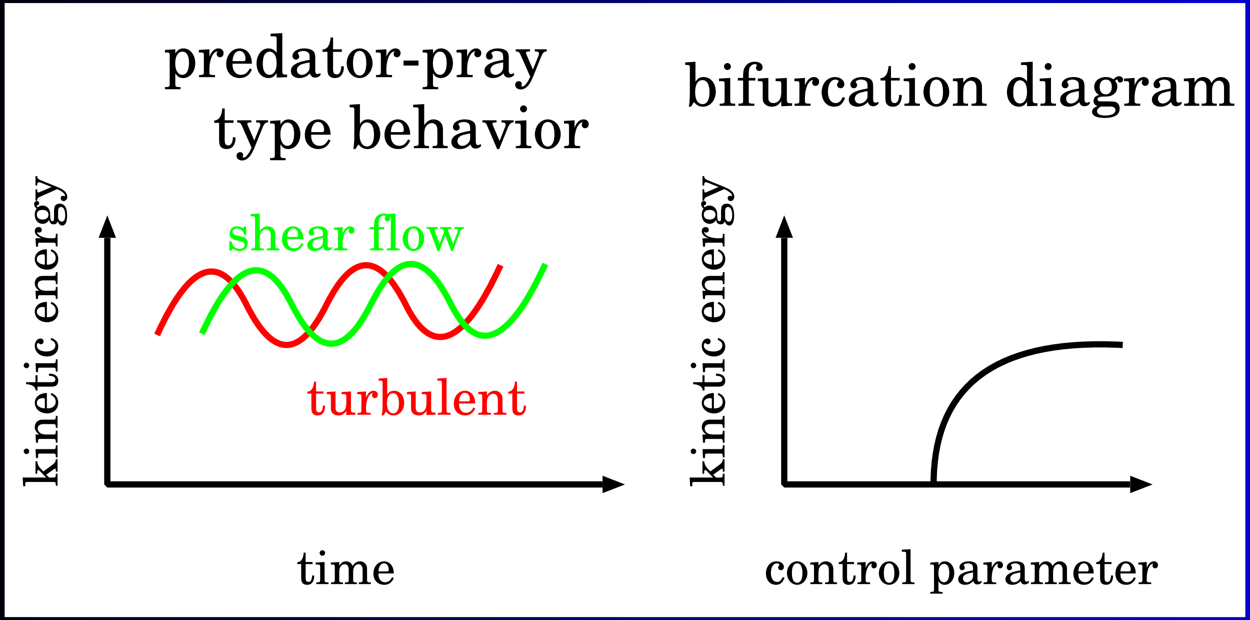
[from http://www.ipp.mpg.de/]



shearing effect by zonal flow

Bifurcation Analysis of Low Dim. Model Predicts Transition

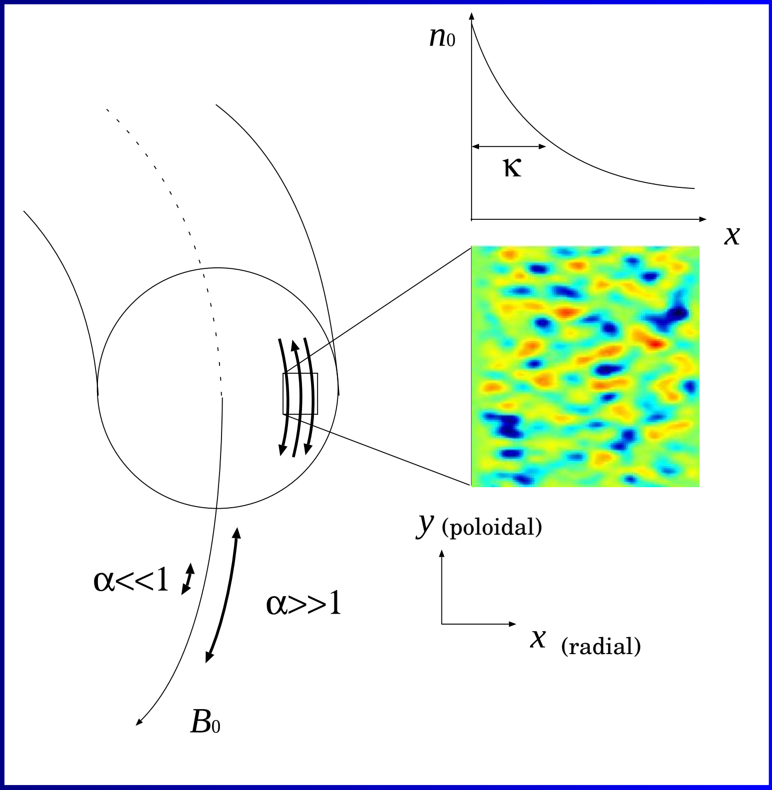
Low dimensional model describes macroscopic system dynamics [Ball *et al*. (2002)]
Interaction between energy variables: shear flow kinetic energy, turbulent kinetic energy, and potential energy



- Model provides economical tool to predict transitions over parameter space
- Requires validation against numerical simulation and/or real experimental data

Hasegawa-Wakatani Model

HW model describes evolution of density fluctuation n and vorticity $\zeta = \nabla^2\varphi$ (φ: electrostatic potential)



$$\begin{aligned}
\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} &= \alpha(\varphi - n) + D_\zeta \nabla^2 \zeta \\
\frac{\partial}{\partial t} n + \{\varphi, n\} &= \alpha(\varphi - n) - \kappa \frac{\partial \varphi}{\partial y} + D_n \nabla^2 n
\end{aligned}$$

$\{a, b\} = \partial a/\partial x \partial b/\partial y - \partial a/\partial y \partial b/\partial x$

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$

D_ζ and D_n are dissipation coefficients

$\kappa \equiv -\partial/\partial x \ln n_0$

$\alpha \equiv \dfrac{T_e k_z^2}{\eta n_0 \omega_{ci} e^2}$: adiabaticity parameter

α

0 Hydrodynamic — Adiabatic ∞ (Hasegawa-Mima)

Modified Hasegawa-Wakatani Model

- Resistive coupling term comes from parallel electron response $\partial j_z/\partial z = 1/\eta \partial^2(\varphi - n)/\partial z^2$ (Ohm's Law)
- Zonal components subtracted from resistive coupling term since the zonal components ($k_y = k_z = 0$) do not contribute to this term

$$\alpha(\varphi - n) \longrightarrow \alpha(\tilde{\varphi} - \tilde{n})$$

Non-zonal $\tilde{\cdot}$ and zonal components $\langle \cdot \rangle$

$$\tilde{\varphi} = \varphi - \langle \varphi \rangle, \quad \tilde{n} = n - \langle n \rangle$$

$$\langle f \rangle = \frac{1}{L_y} \int f \mathrm{d}y \quad (f = \varphi \text{ or } n)$$

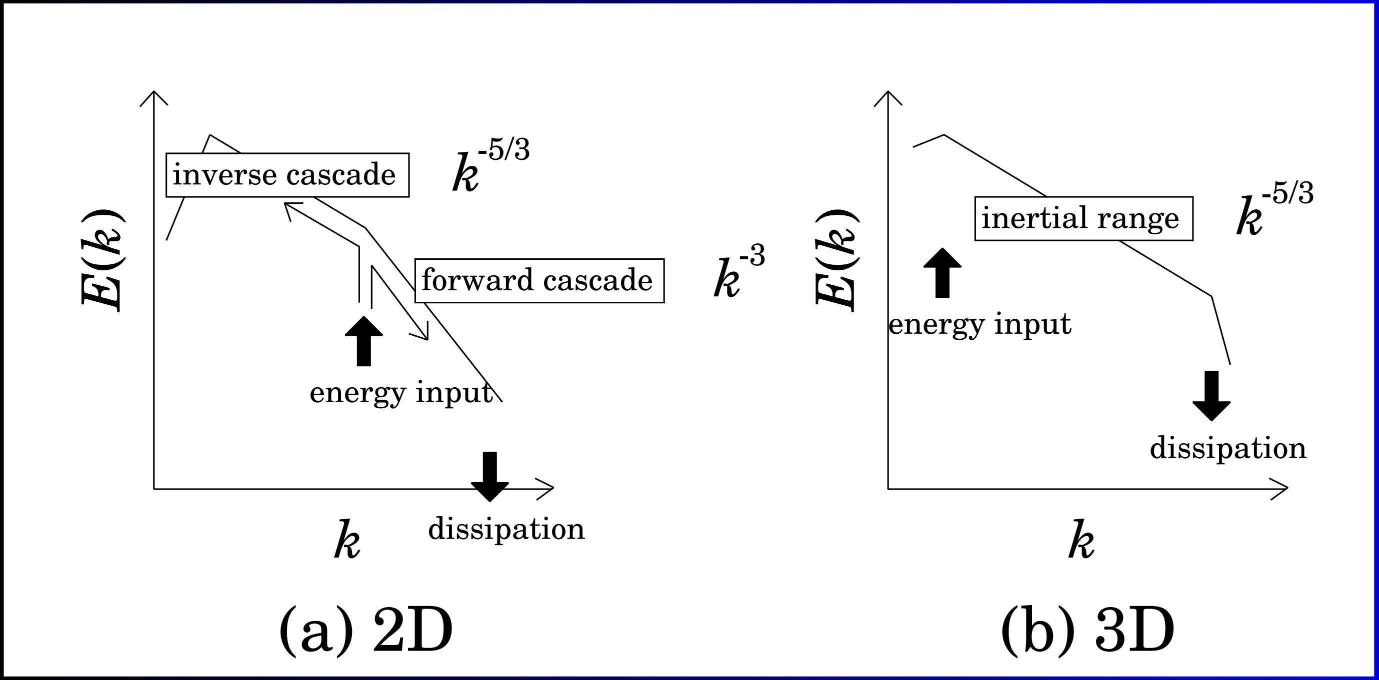
Modified HW model

$$\begin{aligned} \frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} &= \alpha(\tilde{\varphi} - \tilde{n}) + D_\zeta \nabla^2 \zeta \\ \frac{\partial}{\partial t} n + \{\varphi, n\} &= \alpha(\tilde{\varphi} - \tilde{n}) - \kappa \frac{\partial \varphi}{\partial y} + D_n \nabla^2 n \end{aligned}$$

Dual Cascade Yields Large Scale Structure

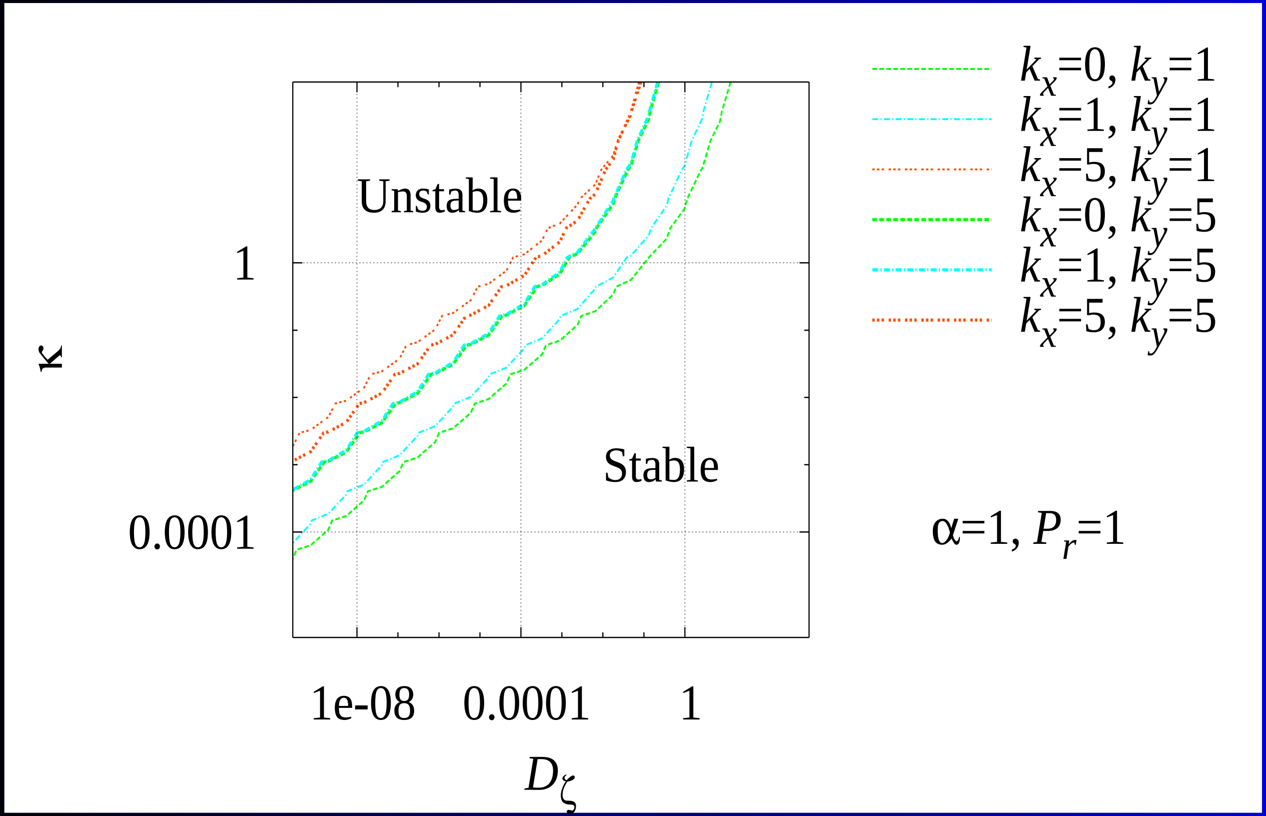
- In 2D, system has two dynamical invariants: total energy E, and generalized enstrophy W ($\sim$ total vorticity)
- In 3D, only E conserved

$$E = \frac{1}{2}\int (n^2 + |\nabla\varphi|^2)\mathrm{d}\boldsymbol{x}, \quad W = \frac{1}{2}\int (n-\zeta)^2 \mathrm{d}\boldsymbol{x}$$



Stability Diagram Provides Indication of Transition Points

Stability threshold in D_ζ (dissipation) – κ (drive) space



- Strong drive (large κ) causes strong instability, which can be stabilized by strong dissipation (large D_ζ)

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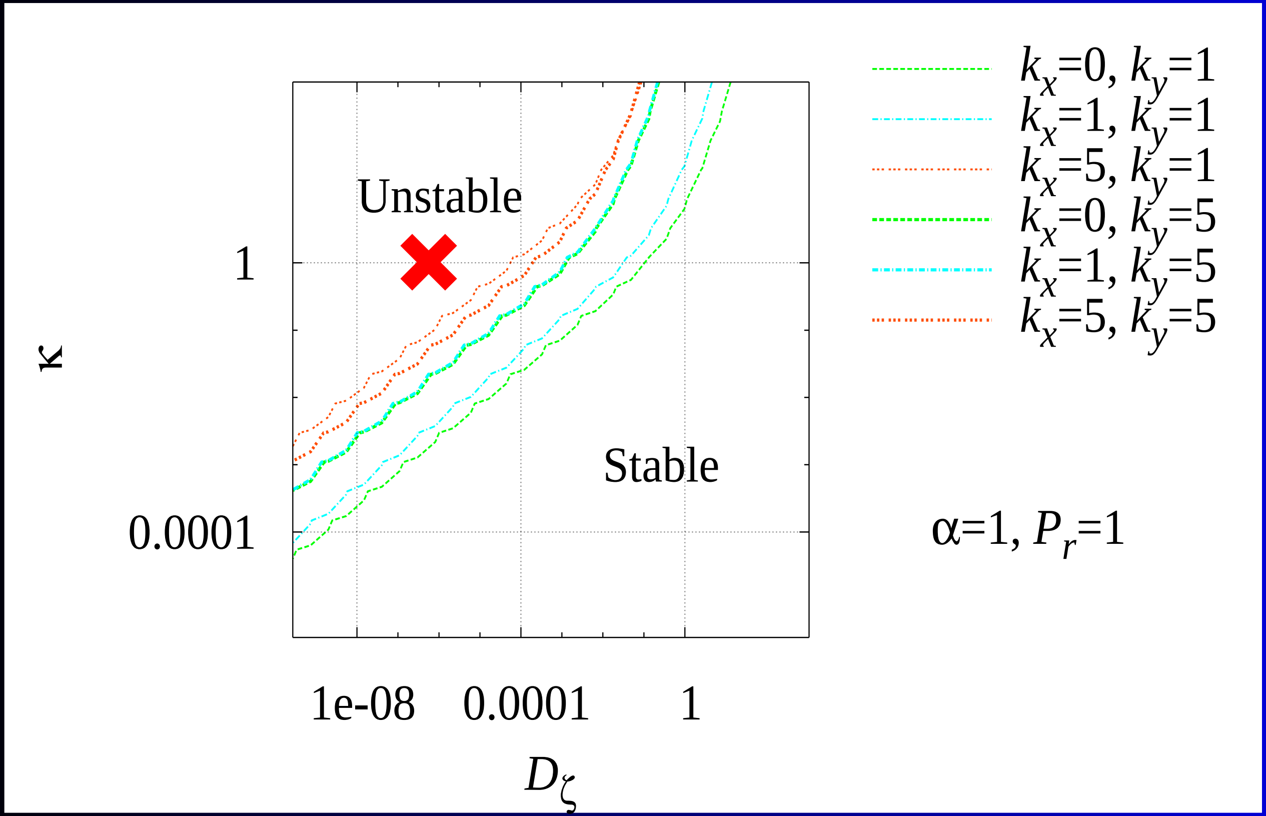

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Algorithm to Solve MHW Model

- Numerical simulation solves MHW model in the slab geometry
- Box size L, determined by smallest wavenumber $\Delta k = 0.15$ [$(2L)^2 = (2\pi/\Delta k)^2$]
- Periodic boundary in y direction; periodic or Dirichlet boundary in x direction eg. Dirichlet condition in x

$$\varphi(x=\pm L, y) = 0, \quad n(x=\pm L, y) = 0, \quad \zeta(x=\pm L, y) = 0$$

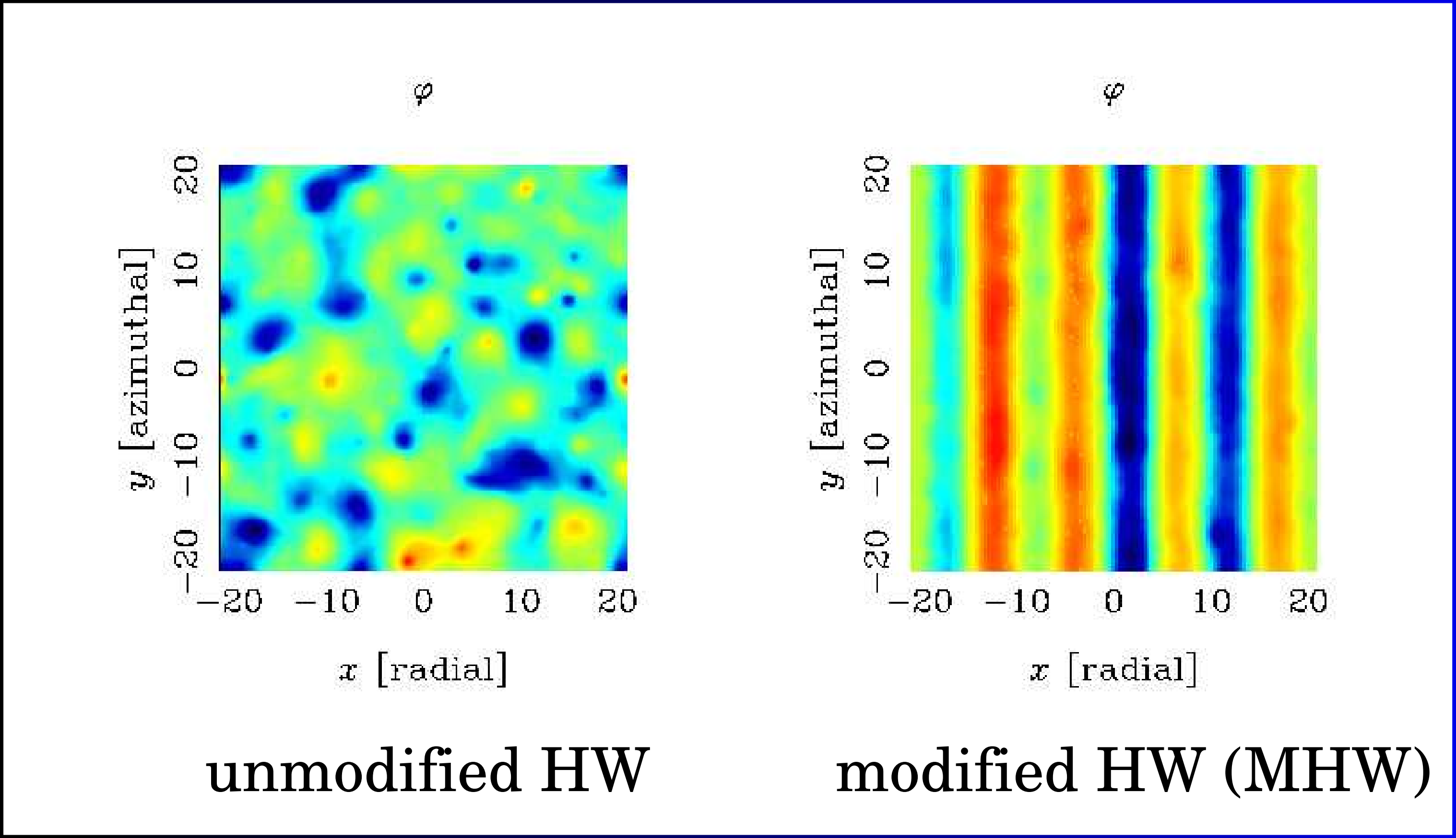
- Time stepping algorithm is a 3rd order explicit linear multistep method. The method for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = \boldsymbol{f}(t, \boldsymbol{x})$ is expressed by

$$\frac{11}{6}\boldsymbol{x}_n - 3\boldsymbol{x}_{n-1} + \frac{3}{2}\boldsymbol{x}_{n-2} - \frac{1}{3}\boldsymbol{x}_{n-3} = 3\boldsymbol{f}(t_{n-1}, \boldsymbol{x}_{n-1}) - 3\boldsymbol{f}(t_{n-2}, \boldsymbol{x}_{n-2}) + \boldsymbol{f}(t_{n-3}, \boldsymbol{x}_{n-3})$$

- Finite difference method is used for spatial discretization
- Poisson bracket term evaluated by the Arakawa's method (Arakawa (1966))

Zonal Structure Generated in MHW Model

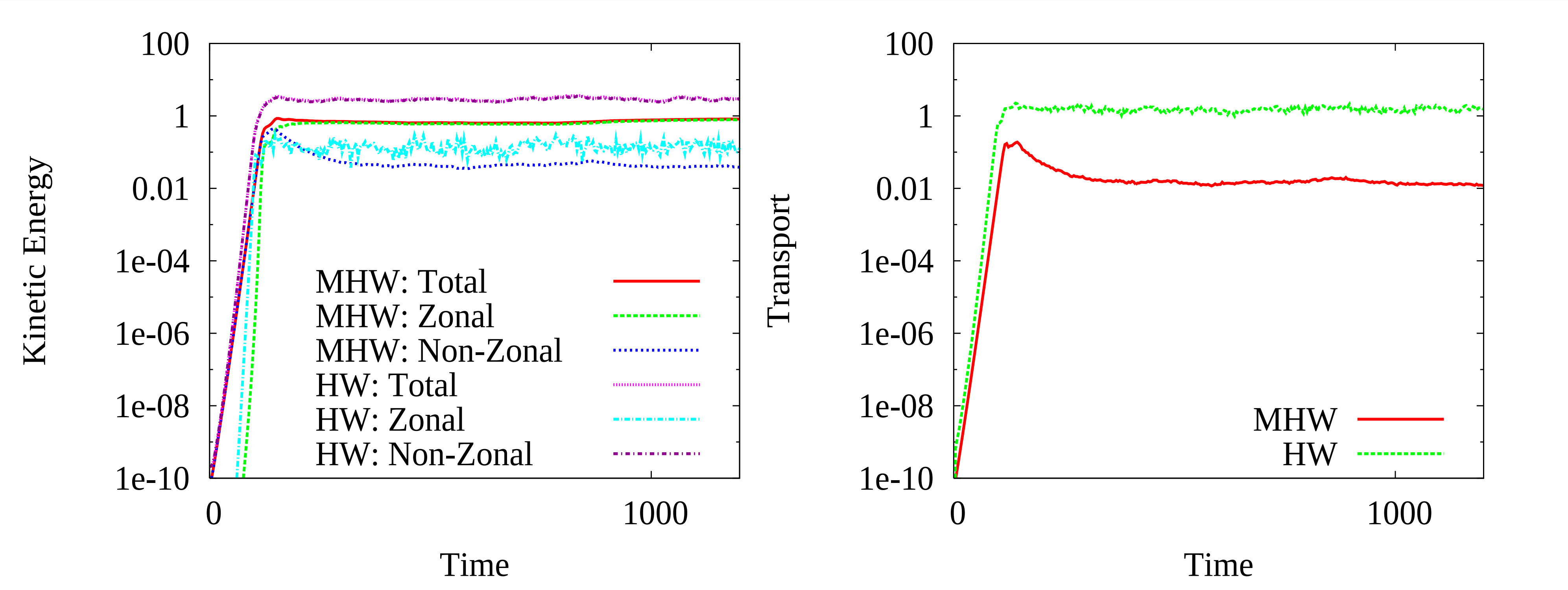
Saturated state of electrostatic potential φ



- Zonally elongated structure is clearly seen in MHW mode

Zonal Flow Suppresses Transport

Time evolution of kinetic energy and transport for modified and unmodified HW model



- Zonal flow components dominates kinetic energy in MHW model
- Once zonal flow is generated, transport level is significantly suppressed

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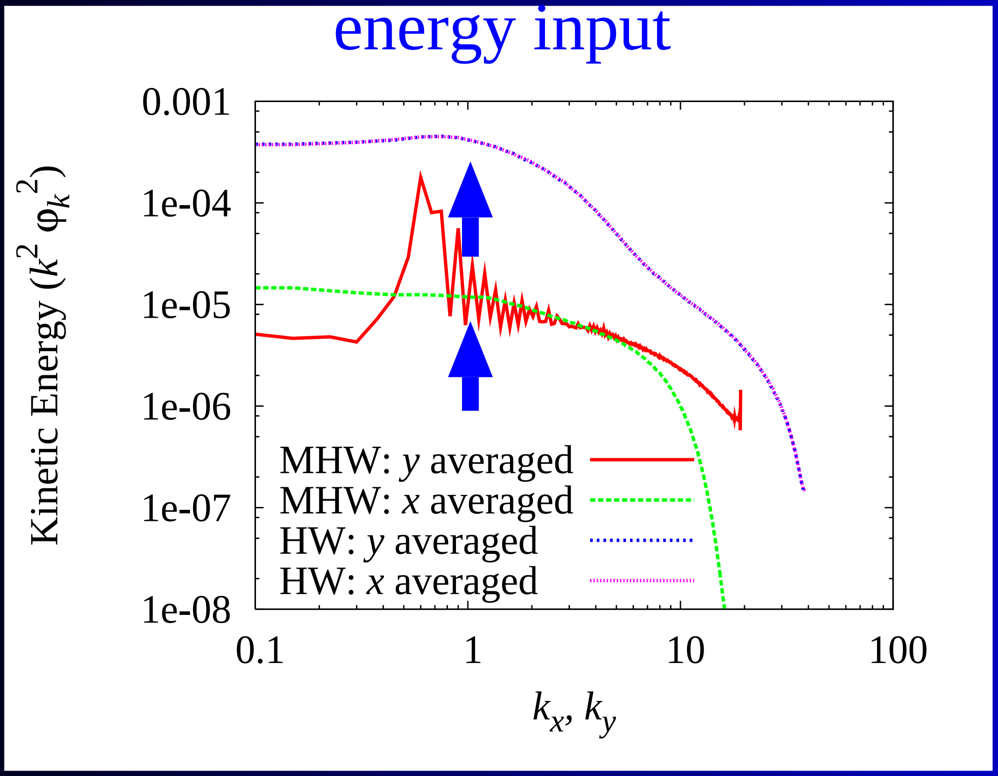
Anisotropy and Inverse Energy Cascade in MHW Model



- Average of spectra defined by $1/K_x \int_0^{K_x} E(k_x, k_y)\mathrm{d}k_x$, $1/K_y \int_0^{K_x} E(k_x, k_y)\mathrm{d}k_y$
- Highly anisotropic spectra is visible for MHW
- Energy inversely cascade to low wavenumber region for MHW

Summary and Conclusion

- High confinement mode crucial to realize fusion power
- Dynamics of L-H transition is not well understood
- Dynamics of L-H transition explored by interplay between direct numerical simulation and low dimensional dynamical model
- Key ingredient is an interaction between large scale zonal flow and turbulent fluctuations
- Succeeded in producing a zonal flow by a simple electrostatic 2D slab resistive drift wave model
- Next: proceed to a systematic survey in parameter space to obtain phase space diagram of transition
- http://wwwrsphysse.anu.edu.au/˜num105/