Bifurcation structure in resistive drift wave turbulence

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Introduction

Fusion plasmas and other turbulent flows in two dimensional (2D) geometry can undergo a spontaneous transition to a turbulence suppressed regime. In plasmas such transitions dramatically enhance the confinement and are known as L-H transitions. From theoretical and experimental work, it is now widely believed that generation of stable coherent structures such as shear flows suppresses cross-field turbulent transport and leads to the confinement improvement. In 2D plasmas and fluids, the net upscale energy flux from small scale turbulent modes to create large scale coherent structures can dominate the classical Kolmogorov cascade to dissipative scales. Recently, a low-dimensional dynamical model for L-H transition has been suggested and analyzed using bifurcation and singularity theories[1]. The model consists of three macroscopic energy variables and, when validated against numerical and/or real experimental data, will provide an economical tool to predict transitions over a parameter space.

In this work we report the results of numerical simulations that both complement the low-dimensional modeling results and raise some interesting issues in their own right. We focus on a model for electrostatic resistive drift wave turbulence, the Hasegawa-Wakatani (HW) model[2], and solve the equations by direct numerical simulation in 2D slab geometry. The HW model has been widely used to investigate anomalous edge transport due to collisional drift waves. Moreover, the model is also capable for the self-organization of a shear flow. Thus we consider the HW model is a good starting point for studying self-consistent turbulence–shear flow interactions, even though it does not describe physics that can be important in specific situations, such as magnetic curvature, magnetic shear, and electromagnetic effect.

Modified Hasegawa-Wakatani Model

The physical setting of the HW model may be considered as the edge region of a tokamak plasma of nonuniform density $n_0 = n_0(x)$ and in a constant equilibrium magnetic field $\boldsymbol{B} = B_0 \nabla z$. Following the drift wave ordering[3], the ion vorticity $\zeta \equiv \nabla^2 \varphi$ (φ is the electrostatic potential, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the 2D Laplacian) and the density fluctuation n are governed by the modified HW (MHW) equations,

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\tilde{\varphi} - \tilde{n}) - D_\zeta \nabla^4 \zeta, \tag{1}$$

$$\frac{\partial}{\partial t}n + \{\varphi, n\} = \alpha(\tilde{\varphi} - \tilde{n}) - \kappa\frac{\partial \varphi}{\partial y} - D_n \nabla^4 n, \tag{2}$$

where $\{a,b\} \equiv \partial a/\partial x \partial b/\partial y - \partial a/\partial y \partial b/\partial x$ is the Poisson bracket. D_ζ and D_n are the dissipation coefficients, respectively. The background density is assumed to have an unchanging exponential profile: $\kappa \equiv -(\partial/\partial x)\ln n_0 = \text{const.}$ $\alpha \equiv -T_\text{e}/(\eta n_0 \omega_\text{ci} e^2)\partial^2/\partial z^2$ is the adiabaticity parameter describing the electron response parallel to the background magnetic field (T_e: electron temperature, η: parallel resistivity, $\omega_\text{ci} \equiv eB_0/m$: ion cyclotron frequency, m: ion mass, e: elementary charge.) In a 2D setting the coupling term operator α becomes a constant coefficient, or parameter, by the replacement $\partial/\partial z \to \mathrm{i}k_z$. As was pointed out recently[4], this resistive coupling term operates only on the non-zonal components defined by

$$\text{non-zonal: } \tilde{f} = f - \langle f \rangle, \quad \text{zonal: } \langle f \rangle = \frac{1}{L_y}\int f \mathrm{d}y, \tag{3}$$

where L_y is the periodic length in y direction. The modification from the original HW model is attributed to this treatment: $\alpha(\varphi - n) \to \alpha(\tilde{\varphi} - \tilde{n})$. Variables have been normalized by

$$x/\rho_\text{s} \to x, \quad \omega_\text{ci}t \to t, \quad e\varphi/T_\text{e} \to \varphi, \quad n_1/n_0 \to n, \tag{4}$$

where $\rho_\text{s} \equiv \sqrt{T_\text{e}/m}\,\omega_\text{ci}^{-1}$ is the ion sound Larmor radius, n_1 is the fluctuating part of the density.

For finite α, the system is unstable against the drift wave instability. The dispersion relation is given by,

$$\omega^2 + \mathrm{i}\omega(b + (1+P_r^{-1})k^4 D_\zeta) - \mathrm{i}b\omega_* - \alpha k^2(k^2 + P_r^{-1})D_\zeta - k^8 P_r^{-1} D_\zeta^2 = 0, \tag{5}$$

where we defined $k^2 = k_x^2 + k_y^2$, $b \equiv \alpha(1+k^2)/k^2$, the drift frequency $\omega_* \equiv k_y\kappa/(1+k^2)$, and the Prandtl number $P_r \equiv D_\zeta/D_n$. If D_ζ and/or D_n is finite, there exists a parameter region where the drift wave instability is suppressed. The stability boundary in the α–κ space for fixed $D_\zeta = 10^{-6}$ and $P_r = 1$ is shown in Fig.1. If we choose the

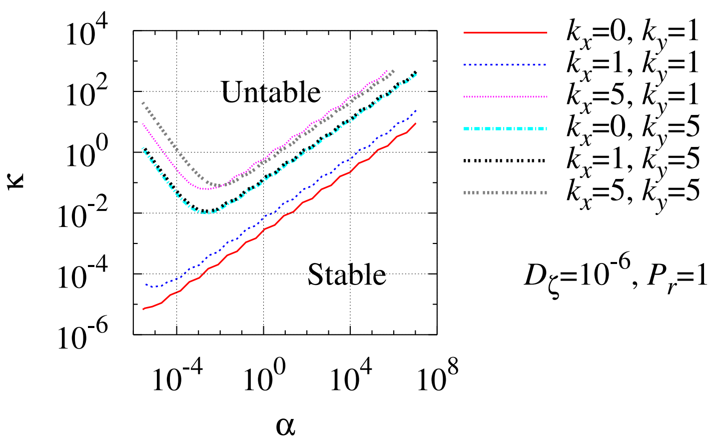

Figure 1: Linear stability boundary in α–κ space.

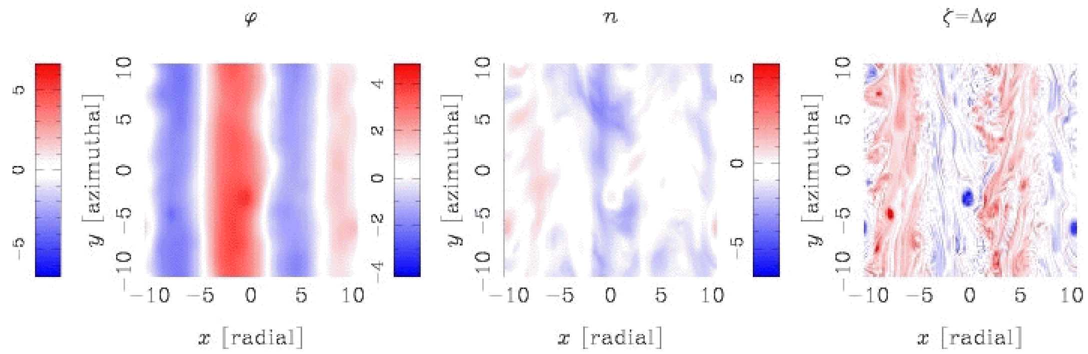

Figure 2: Contour plot of φ, n, and ζ in the saturated state. Zonally elongated structure of the electrostatic potential is clearly visible.

parameters in the unstable region, initially small random perturbations grow linearly until the nonlinear terms begin to dominate. Then, the system arrives at a nonlinearly saturated state where the energy input from the background density profile and the energy output due to the parallel resistivity and the dissipation balance.

Simulation Results

The HW equations are solved in a double periodic slab domain with box size $(2L)^2 = (2\pi/\Delta k)^2$ where the lowest wavenumber $\Delta k = 0.3$. The equations are discretized on 256×256 grid points by the finite difference method. Arakawa's method is used for evaluation of the Poisson bracket. Time stepping algorithm is the third order explicit linear multistep method.

Figure 2 shows the saturated state of variables. We can observe that the zonally elongated structure of the electrostatic potential is generated. Once the zonal flow is generated, the cross-field transport $\Gamma_n = -\kappa \int n(\partial\varphi/\partial y)\mathrm{d}x\mathrm{d}y$ is significantly suppressed. The build of the zonal flow, and resulting transport suppression in the MHW model highlight the difference from the original HW model[5].

In Fig. 3, we plot the ratio of the kinetic energy of the zonal flow ($F \equiv 1/2 \int (\partial \langle\varphi\rangle/\partial x)^2 \mathrm{d}x\mathrm{d}y$) to the total kinetic energy ($E \equiv 1/2 \int |\nabla\varphi|^2 \mathrm{d}x\mathrm{d}y$) against κ and α. If we fix α and increase κ, the zonal flow dominant saturated state suddenly jumps to the zonal flow suppressed state where rather isotropic vortices appear at $\kappa \sim 10$. On the other hand a transition from the zonal flow suppressed state to zonal flow dominant state occurs with increasing α and fixed κ at $\alpha \sim 0.5$. If we map the bifurcation diagram obtained in Fig.3, we can recognize that the zonal flow suppressed (or turbulent dominant) states correspond to the linearly unstable region.

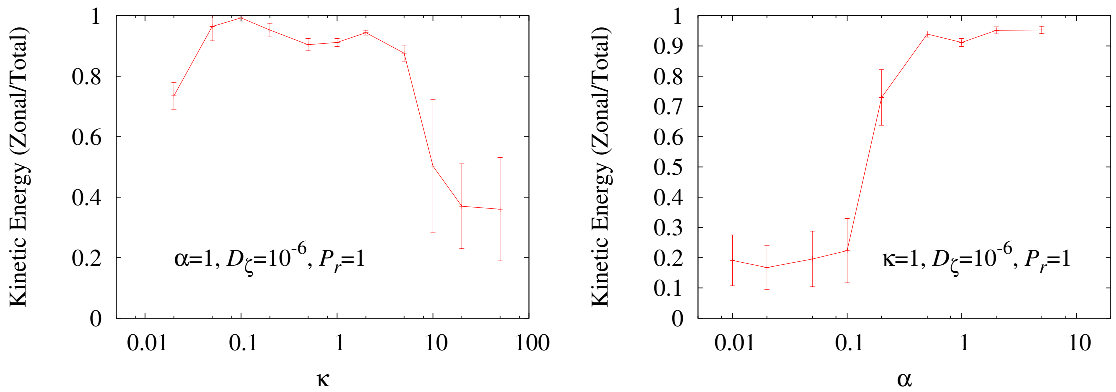

Figure 3: Parameter dependence of the ratio of the zonal kinetic energy to the total kinetic energy. Transitions from a low zonal flow to a high zonal flow state occur.

Conclusion

We have performed numerical simulations of the MHW model to study bifurcation structures in a parameter space. We have shown that, in the MHW model, zonal flows are self-organized, and turbulence transport is suppressed due to the zonal flows. Sudden transitions from the zonal flow dominant state to the zonal flow suppressed state are observed if we control parameters (the strength of the linear drive κ, or the parallel electron adiabaticity α) to the direction to which the system become more unstable.

The correspondence of the bifurcation diagram obtained from the numerical simulations to that obtained from the low-dimensional dynamical model will be considered.

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