Collisionless and Collisional Tearing Mode in Gyrokinetics

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Introduction

- Magnetic reconnection is ubiquitous in fusion and astrophysical plasmas, which allows topological change of field lines, and convert field energy into plasma flow and heat. A detailed understanding of the phenomena in collisionless (kinetic) regime is still missing. Our goal is to provide comprehensive picture of the magnetic reconnection phenomena using kinetic model.

- We present a comprehensive linear study (though our ultimate goal is to understand nonlinear evolution) of the parameter space covering both the collisional and the collisionless regimes in a strong guide magnetic field limit using AstroGK[1] astrophysical gyrokinetics code. Recently implemented model collision operator enables to simulate the collisionless and collisional regimes and their intermediate seamlessly.

- Primary aim of this study is to trace from macroscopic MHD scale down to electron inertial scale (d_e) by changing collisionality. We may pass through two-fluid (d_i) regime, kinetic ion regime (ρ_i or ρ_s), and then reach at collisionless regime where the electron inertia mediates magnetic reconnection process.

- Only the kinetic description can capture these collisionless effects acurately. This study also enables to identify the regions of validity of various fluid modelings.

Gyrokinetics

Gyrokinetic equation describes evolutions of distribution function h in gyrocenter coordinate $(\boldsymbol{R},\boldsymbol{V}),\ \boldsymbol{R}=\boldsymbol{r}+\frac{\boldsymbol{v}\times\boldsymbol{b}_0}{\Omega}$

$$f_s=f_{0s}+\delta f_s=f_{0s}-\frac{q_s\phi}{T_{0s}}f_{0s}+h(\boldsymbol{R})$$

$$\frac{\partial h_s}{\partial t}+\frac{1}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s},h_s\}=\frac{q f_{0s}}{T_{0s}}\frac{\partial\langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t}+\langle C(h_s)\rangle_{\boldsymbol{R}_s},\qquad \chi=\phi-\boldsymbol{v}\cdot\boldsymbol{A}.$$

Field equations relate between fields and moments in $\boldsymbol{r}$

$$\sum_s q_s\tilde{n}_s=0,\qquad \tilde{n}_s=\int\delta f_s\,\mathrm{d}\boldsymbol{v}=\int\langle h_s\rangle_{\boldsymbol{r}}\mathrm{d}\boldsymbol{v}-\frac{q_s n_{0s}}{T_{0s}}\phi$$

$$\nabla_\perp^2A_\parallel=-\mu_0\sum_s q_s n_{0s}\tilde{u}_{\parallel,s},\qquad \tilde{u}_{\parallel,s}=\frac{1}{n_{0s}}\int\delta f_s v_\parallel\mathrm{d}\boldsymbol{v}=\frac{1}{n_{0s}}\int\langle h_s v_\parallel\rangle_{\boldsymbol{r}}\mathrm{d}\boldsymbol{v}$$

$$B_0\nabla_\perp\delta B_\parallel=-\mu_0\nabla_\perp\cdot\tilde{\mathsf{P}}_{\perp,s}\qquad \begin{aligned}\tilde{\mathsf{P}}_s&=\int m\boldsymbol{v}\boldsymbol{v}\,\delta f_s\,\mathrm{d}\boldsymbol{v}\\&=\int mv_\parallel^2\langle h_s\rangle_{\boldsymbol{r}}\mathrm{d}\boldsymbol{v}+\int\langle m\boldsymbol{v}_\perp\boldsymbol{v}_\perp h_s\rangle_{\boldsymbol{r}}\mathrm{d}\boldsymbol{v}-q_s n_{0s}\phi\,\mathrm{I}\\&=P_{\parallel\parallel,s}\boldsymbol{e}_\parallel\boldsymbol{e}_\parallel+P_{\perp\perp,s}\boldsymbol{e}_\perp\boldsymbol{e}_\perp+P_{\theta\theta,s}\boldsymbol{e}_\theta\boldsymbol{e}_\theta+P_{\theta\parallel,s}\boldsymbol{e}_\theta\boldsymbol{e}_\parallel\end{aligned}$$

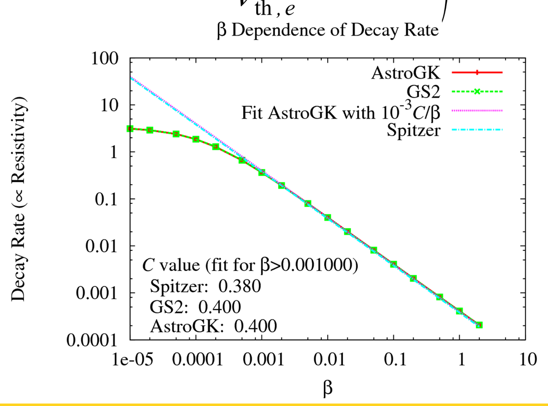
Collision operator

In addition to like-particle collisions (which satisfies physical requirements such as particle, momentum, and energy conservations and Boltzmann's H-theorem), we have implemented electron-ion collisions producing the resistivity[1,2].

$$\langle C_{\mathrm{ei}}(h)\rangle_{\boldsymbol{R}}=\sum_{\boldsymbol{k}}e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}}\nu_{\mathrm{ei}}\left(\frac{v_{\mathrm{th,e}}}{V}\right)^3\left(\frac{1}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial h_{\mathrm{e},\boldsymbol{k}}}{\partial\xi}-\frac{k^2V^2}{4\Omega_{0\mathrm{e}}}(1+\xi^2)h_{\mathrm{e},\boldsymbol{k}}+\frac{2J_0(\alpha_{\mathrm{e}})V_\parallel u_{\parallel,\mathrm{i}}}{v_{\mathrm{th},e}^2}f_{0\mathrm{e}}\right)$$

Spitzer resistivity given by $\eta/\mu_0=m_{\mathrm{e}}/(1.98\tau_{\mathrm{e}}n_e e^2)$, where $\tau_{\mathrm{e}}=3\sqrt{\pi}/4\nu_{\mathrm{ei}}$, leads to resistive current decay rate

$$\tau_{\mathrm{decay}}^{-1}=C\nu_{\mathrm{ei}}(d_{\mathrm{e}}k)^2$$



Numerical test of the e-i collision term confirms that the collision operator really yields Spitzer resistivity with ~5% error.

Fluid theories

1F-MHD (single-fluid)[4,5]

parameters: $\Delta'a=\left[\frac{\mathrm{d}}{\mathrm{d}x}\ln A_\parallel\right],\quad \frac{\tau_{\mathrm{H}}}{\tau_{\mathrm{A}}}=\frac{1}{ka B_0'},\quad S=\frac{\tau_{\mathrm{R}}}{\tau_{\mathrm{A}}}=\frac{\mu_0 a V_{\mathrm{A}}}{\eta}$

assumptions:
- time scale separation $1/\tau_{\mathrm{R}}\ll\gamma\ll1/\tau_{\mathrm{H}}$
- scale separation $l\ll a$ (l is the current layer width)

dispersion relation and current layer scaling:

$$\Delta'a=-\frac{8}{\pi}(\gamma\tau_{\mathrm{A}})^{5/4}\left(\frac{\tau_{\mathrm{H}}}{\tau_{\mathrm{A}}}\right)^{1/2}\left(\frac{\tau_{\mathrm{R}}}{\tau_{\mathrm{A}}}\right)^{3/4}\frac{\Gamma((\lambda^{3/2}-1)/4)}{\Gamma((\lambda^{3/2}+5)/4)},\qquad \lambda=(\gamma\tau_{\mathrm{A}})\left(\frac{\tau_{\mathrm{H}}}{\tau_{\mathrm{A}}}\right)^{2/3}\left(\frac{\tau_{\mathrm{R}}}{\tau_{\mathrm{A}}}\right)^{1/3}$$

$$\frac{l}{a}\sim(\gamma\tau_{\mathrm{A}})^{1/4}\left(\frac{\tau_{\mathrm{H}}}{\tau_{\mathrm{A}}}\right)^{1/2}\left(\frac{\tau_{\mathrm{R}}}{\tau_{\mathrm{A}}}\right)^{-1/4}$$

2F-MHD (two-fluid)[6,7,8]

additional physics: Hall effect, FLR effect, electron inertia

two scale separations:

$$d_\beta=c_\beta d_{\mathrm{i}}=\begin{cases}\rho_{\mathrm{s}} & (\beta\ll1)\\ d_{\mathrm{i}} & (\beta\gg1)\end{cases}\qquad \delta=\sqrt{d_{\mathrm{e}}^2+\frac{a^2}{\tau_{\mathrm{R}}\gamma}}$$

$$c_\beta=\sqrt{\frac{\beta}{2/\kappa+\beta}}$$

$$\rho_{\mathrm{s}}=c_{\mathrm{s}}/\omega_{\mathrm{ci}}=\sqrt{\frac{\kappa(T_{\mathrm{e}}+T_{\mathrm{i}})}{m_{\mathrm{i}}}}/\frac{eB_0}{m_{\mathrm{i}}},$$

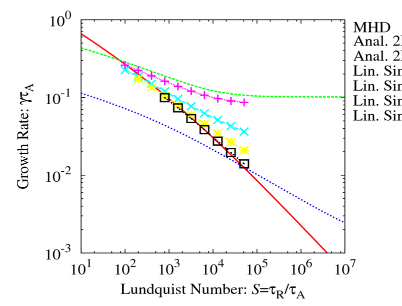
$$d_s=c/\omega_{\mathrm{p}s}\quad(s=\mathrm{i},\mathrm{e})$$

$\kappa=5/3$ is the ratio of the specific heat

dispersion relation and current layer scaling:

$$\frac{Q^2}{G(Q/c_\beta)}\frac{d_\beta}{a}+\frac{2}{\Delta'a}=\frac{2}{\pi Q}\frac{\delta}{a}\qquad Q=\frac{\gamma\tau_{\mathrm{A}}}{d_\beta k}$$

$$\frac{l}{a}\sim\sqrt{2c_\beta^2Q}\frac{\delta}{a}\qquad G(x)=\frac{\sqrt{x}}{2}\frac{\Gamma((x+1)/4)}{\Gamma((x+3)/4)}$$



Linear 2F code result and analytic DRs

Simulation setting

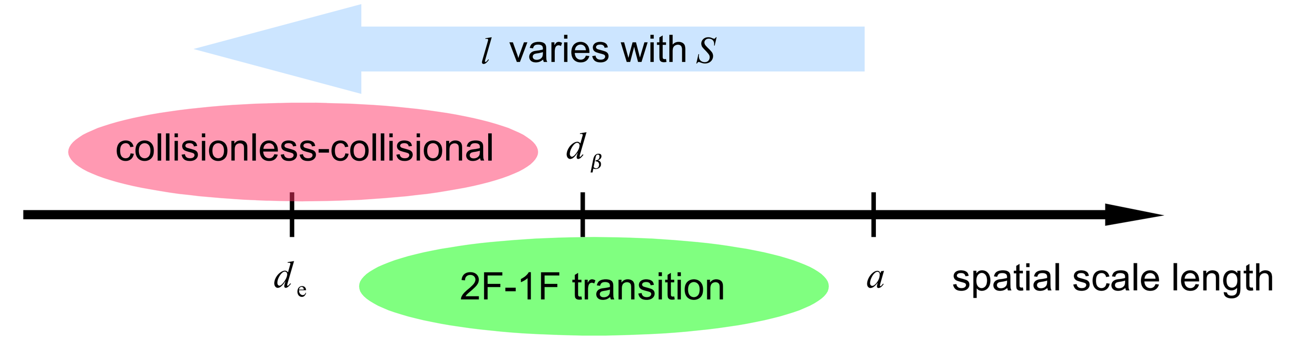
AstroGK solves gyrokinetic equations in periodic slab domain

We assume uniform background ($\nabla n_0=\nabla T_0=\nabla B_0=0$), and $\partial/\partial z=0$

Initial condition: shifted Maxwellian electron ($n_{0\mathrm{e}}=1,T_{0\mathrm{e}}=1,u_{\parallel,\mathrm{e}}$)
non-shifted Maxwellian ion ($n_{0\mathrm{i}}=1,T_{0\mathrm{i}}=1,u_{\parallel,\mathrm{i}}=0$)
electron flow (amplitude and profile) is chosen to give $A_{\parallel,\mathrm{eq}}=\frac{A_{\parallel,0}}{\cosh^2(x/a)}S(x)$ ($S(x)$ is to make periodic)

fixed parameters: $\Delta'a=23.91,\ \tau_{\mathrm{H}}/\tau_{\mathrm{A}}=0.486,\ ka=0.8,\ \nu_{\mathrm{i}}=0$ (inviscid ion)
other parameters (unless otherwise stated): $m_{\mathrm{e}}/m_{\mathrm{i}}=0.01,\ \beta_{\mathrm{e}}=0.3$

Scan collisionality to vary current layer width



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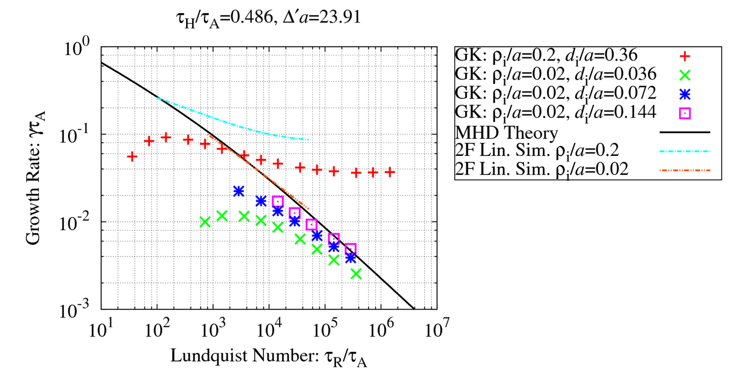
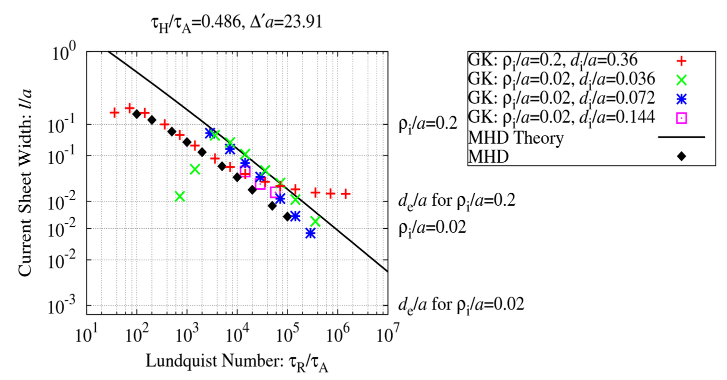
Simulation Result

Growth Rate



Current Layer Width



- Transition to collisionless regime

  As S increases, growth rate becomes independent of S, and current layer width asymptotes to d_{e} scale.

- Tangled ion scales

  Even if we decrease ion scales collectively, we cannot neglect small kinetic effects ($\rho_{\mathrm{i}}=0.02$).

- Release entanglement

  By separating d_{i} scale (low β), we recover 1F-MHD scaling.

- Disagreement with fluid theory

  We have not yet reached agreement with the fluid theory. Analytic DR is not working in this parameter regime (not shown).

Conclusions

We have shown the ability of AstroGK to perform seamless study of widely separated scale hierarchy.

It has captured transitions of collisional - collisionless regimes, and 1F-MHD – 2F-MHD regimes. However, there are still deviations from the fluid theory.

Detailed analysis is necessary to understand the behavior when some scales are comparable to each other.

References

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