Bifurcation in Resistive Drift Wave Turbulence

Complex'07

R. Numata, R. Ball, R.L. Dewar, L. Stals*

Dept. of Theoretical Physics, Research School of Phys. Sci. & Eng.,

The Australian National University

*)Dept. Mathematics, The Australian National University

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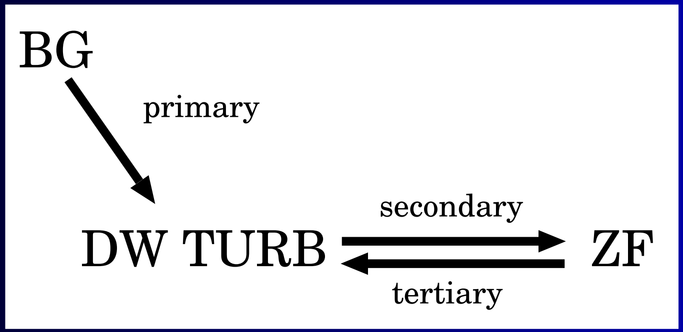
Transition in Fusion Plasmas

- In many magnetically confined fusion experiments, plasmas may undergo a spontaneous transition to a turbulence suppressed regime, which is known as *L-H* (low- to high- confinement) transition
- *L-H* transition is characterized by steep gradients in density and pressure at edge region.
- Zonal flows play key roles in *L-H* transition
- 3 key processes are generation of turbulence by drift wave (primary instability), self-organization of zonal flow (secondary instability), and destabilization of zonal flow (tertiary instability)
- 3 energetic subsystems involved: Background potential energy, Turbulence kinetic energy, Zonal flow kinetic energy.

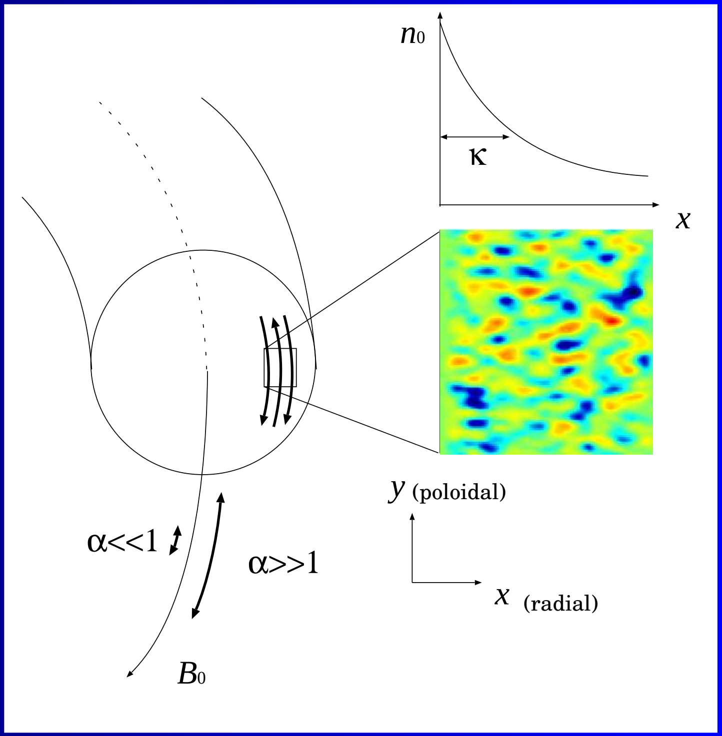


Hasegawa-Wakatani Model

HW model describes evolution of density fluctuation n and vorticity $\zeta = \nabla^2\varphi$ (φ: electrostatic potential)

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\varphi - n) - D(-\nabla^2)^m \zeta$$
$$\frac{\partial}{\partial t}n + \{\varphi, n\} = \alpha(\varphi - n) - \kappa\frac{\partial\varphi}{\partial y} - D(-\nabla^2)^m n$$

$\{a, b\} = \partial a/\partial x \partial b/\partial y - \partial a/\partial y \partial b/\partial x$
$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$
D is dissipation coefficient,
$\kappa \equiv -\partial/\partial x \ln n_0$
$\alpha \equiv \frac{T_e k_z^2}{\eta n_0 \omega_{ci} e^2}$: adiabaticity parameter



Modified Hasegawa-Wakatani Model

- Resistive coupling term comes from parallel electron response $\partial j_z/\partial z = 1/\eta \partial^2(\varphi - n)/\partial z^2$ (Ohm's Law)
- Zonal components subtracted from resistive coupling term since the zonal components ($k_y = k_z = 0$) do not contribute to this term [Hammett *et al* (1993)]

Modified HW model

$$\begin{aligned}\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} &= \alpha(\tilde{\varphi} - \tilde{n}) - D(-\nabla^2)^m \zeta \\ \frac{\partial}{\partial t} n + \{\varphi, n\} &= \alpha(\tilde{\varphi} - \tilde{n}) - \kappa \frac{\partial \varphi}{\partial y} - D(-\nabla^2)^m n\end{aligned}$$

where non-zonal $\tilde{\cdot}$ and zonal components $\langle \cdot \rangle$

$$\tilde{\varphi} = \varphi - \langle \varphi \rangle, \quad \tilde{n} = n - \langle n \rangle, \quad \langle f \rangle = \frac{1}{L_y} \int f \mathrm{d}y \quad (f = \varphi \text{ or } n)$$

Parallel wave number should be chosen to give maximum growth rate: $\alpha = 4k^2 k_y \kappa/(1+k^2)^2$ []Hasegawa & Wakatani (1984)]

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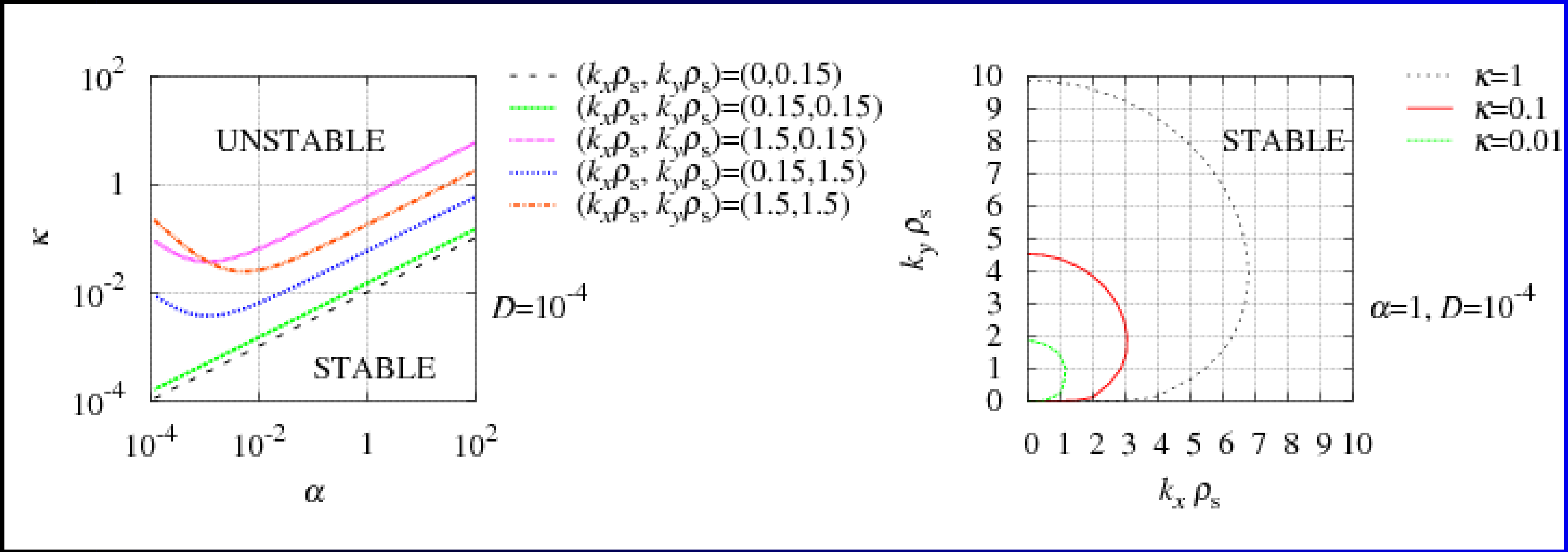
Stability Diagram Provides Indication of Transition Points

Stability threshold in α (electron adiabaticity) – κ (drive) space
There exists one linearly growing mode if $D = 0$, which can be stabilized by finite D.



- Large $\kappa \longrightarrow$ destabilizing, Large $\alpha \longrightarrow$ stabilizing.
- Growth rate monotonically decreases with k_x, peaks at $k_y \sim 1$.

Algorithm to Solve MHW Model

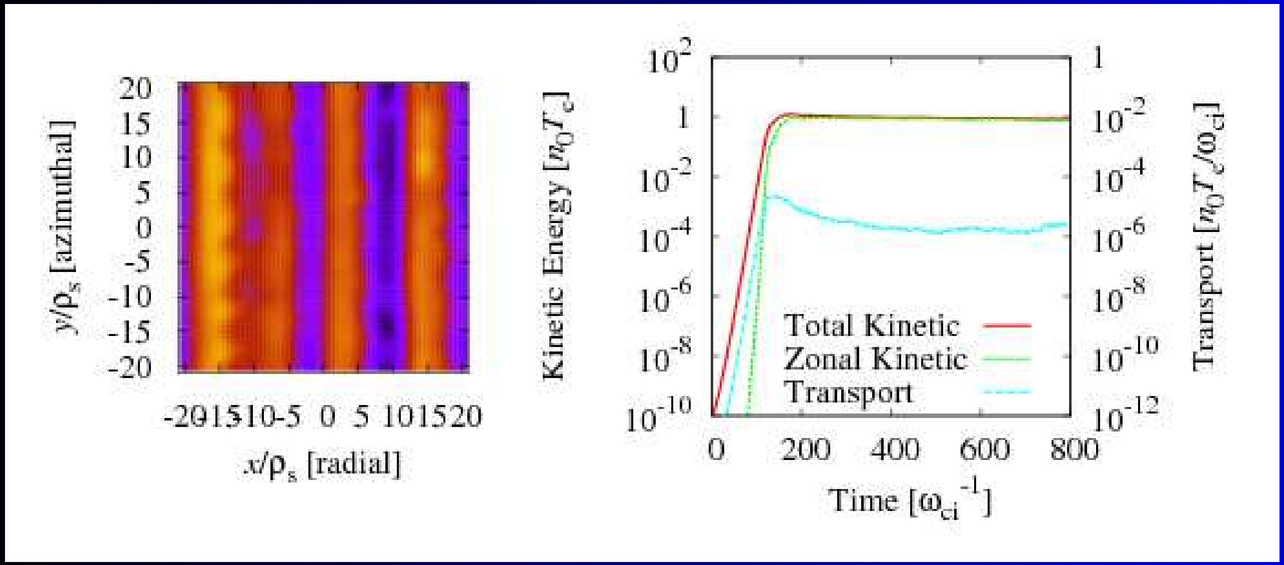
The code is originally developed by B. Scott, IPP.

- MHW model is solved in the slab geometry
- Box size L, determined by smallest wavenumber $\Delta k = 0.15$ [$L = 2\pi/\Delta k$]
- Periodic boundary in both x and y direction
- Time stepping algorithm is a 3rd order explicit linear multistep method (stiffly stable method)]. The method for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = \boldsymbol{f}(t, \boldsymbol{x})$ is expressed by

$$\frac{11}{6}\boldsymbol{x}_n - 3\boldsymbol{x}_{n-1} + \frac{3}{2}\boldsymbol{x}_{n-2} - \frac{1}{3}\boldsymbol{x}_{n-3} = 3\boldsymbol{f}(t_{n-1}, \boldsymbol{x}_{n-1}) - 3\boldsymbol{f}(t_{n-2}, \boldsymbol{x}_{n-2}) + \boldsymbol{f}(t_{n-3}, \boldsymbol{x}_{n-3}$$

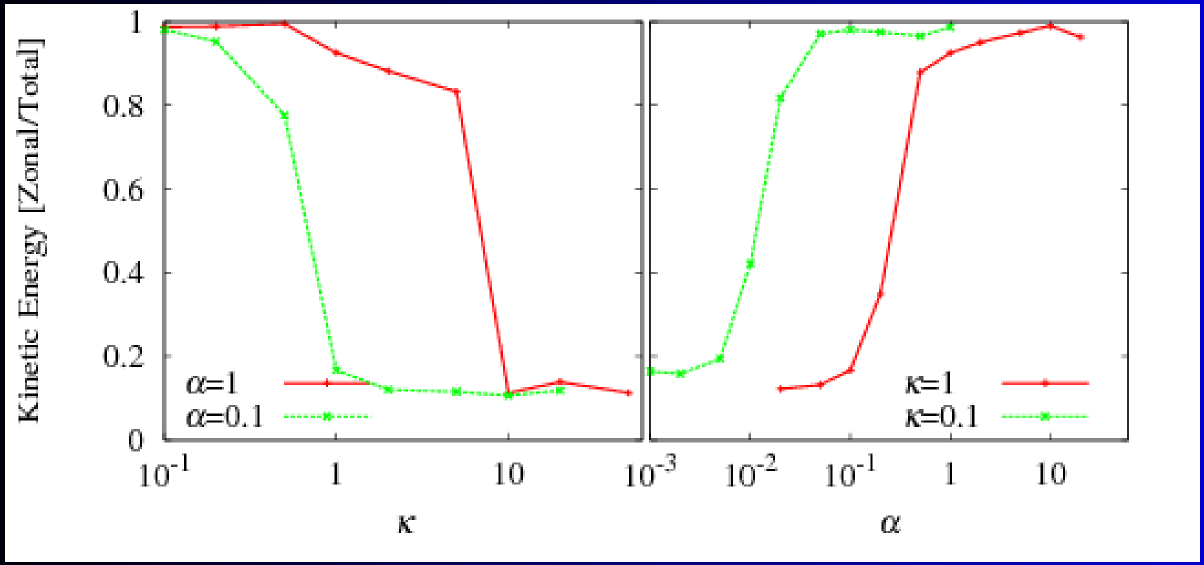
- Finite difference method is used for spatial discretization
- Poisson bracket term evaluated by the Arakawa's method (Arakawa (1966))
- MPI parallelized to implement on APAC-NF SGI Altix 3700 Bx2 cluster

Zonal Flow Reduces Transport



- Parameters: $\kappa = 1$, $\alpha = 1$, $D = 10^{-4}$
- Contour plot of electrostatic potential shows generation of counter streaming zonal flows
- Most of kinetic energy is contained in zonal flow
- Once zonal flow is generated, cross-field transport is significantly suppressed

Transition to Trubulence Observed



- Zonal flow kinetic energy: $F \equiv 1/2 \int \left(\frac{\partial\langle\varphi\rangle}{\partial x}\right)^2 \mathrm{d}x\mathrm{d}y$
- Sudden transition from zonal flow dominated state to turbulence dominated state occurs
- κ destabilize zonal flows
- α stabilizing: adiabatic regime is stable

Stability of Zonal flow

Zonal flow may be subject to Kelvin-Helmholtz (K-H) instability

We assume $\varphi = \varphi_0(x) + \hat{\varphi}(x)e^{\mathrm{i}(k_y y - \omega t)}$, $n = \hat{n}(x)e^{\mathrm{i}(k_y y - \omega t)}$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2 + \frac{k_y V''(x)}{\omega - k_y V(x)} - \frac{\mathrm{i}\alpha}{\omega - k_y V(x) + \mathrm{i}\alpha}\left(1 - \frac{k_y \kappa}{\omega - k_y V(x)}\right)\right]\hat{\varphi}(x) = 0 \qquad (1)$$

$$\hat{n}(x) = \frac{\mathrm{i}\alpha + \kappa}{\omega - k_y V(x) + \mathrm{i}\alpha}\hat{\varphi}(x) \qquad (2)$$

Flow profile

$$\partial\varphi_0/\partial x = V_0 \sin(\lambda x)$$

$\lambda = n_\lambda \pi / L$ (L: box size), $n_\lambda = 4$

Eigenvalue problem is solved numerically by shooting method

Fixed boundary condition is assumed for simplicity

Stability Condition in Two Limits

Tollmien proved existence of marginally stable eigenfunction satisfying ($\omega_{\rm s}/k_{\rm s} = V(x_{\rm s})$) where $x_{\rm s}$ is the inflection point [Tollmien 1935]

- Hydrodynamic Limit [$\alpha \to 0$]

$$k_{\rm s,0} = \sqrt{\lambda^2 - \left(\frac{n\pi}{L}\right)^2} \Longrightarrow \lambda > \frac{\pi}{L} \tag{3}$$

- Adiabatic Limit [$\alpha \to \infty$]

$$k_{\rm s,\infty} = \sqrt{\lambda^2 - \left(\frac{n\pi}{L}\right)^2 + 1} \Longrightarrow \lambda > \sqrt{\left(\frac{\pi}{L}\right)^2 - 1} \tag{4}$$

Zonal flows obtained in numerical simulation typically have $\lambda \sim 0.3$, which are unstable

in hydrodynamic limit, and stable in adiabatic limit

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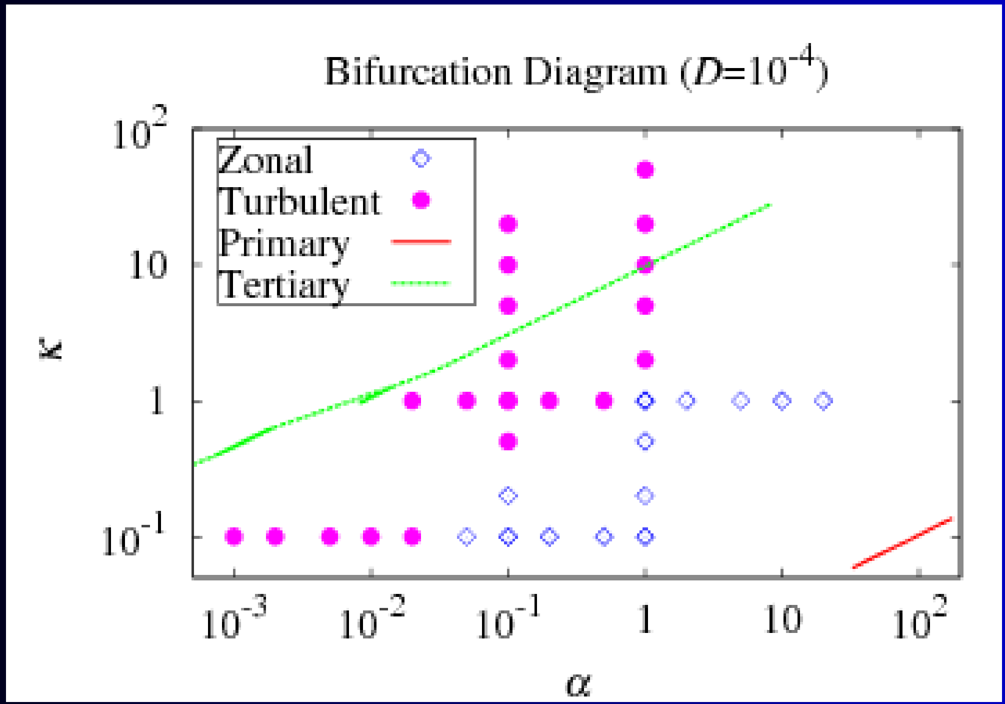
Bifurcation Diagram



- Upshift of onset of turbulence [*Dimits Shift*]
- Instability of zonal flow is underestimated
  - Boundary Condition
  - Viscosity [singular perturbation]

Summary and Conclusion

- We have performed simulations of modified Hasegawa-Wakatani model
- Zonal flows are generated, and the zonal flows suppresses cross-field turbulent transport in MHW model
- Transition from zonal flow dominant to zonal flow suppressed state is observed
- Stability of zonal flow against Kelvin-Helmholtz instability is studied, and compared with numerical results
- Upshift of onset of turbulence in parameter space is found
- Discrepancy between KH analysis and simulation results may be ascribed to simplification of KH analysis, and/or accuracy of transition detection by numerical simulation

Hasegawa-Wakatani model is particulary simple model, but includes enough physics to analyze the interplay between zonal flow and turbulence which brings transition observed in fusion plasmas. This model exhibits more interesting phenomena, e.g. oscillatory behavior like predator-prray model.