Bifurcation Structure in Resistive Drift Wave Turbulence

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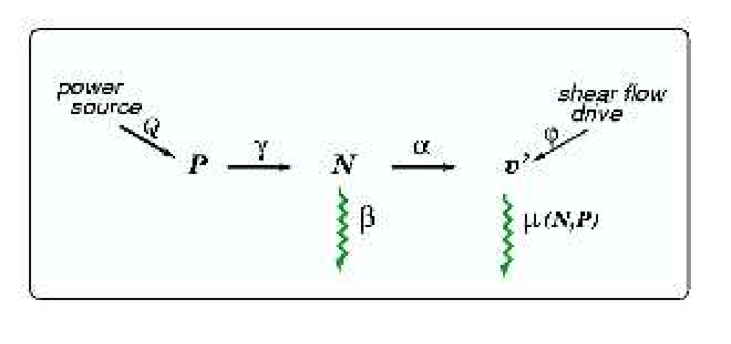
Bifurcation Analysis in Plasma Turbulence

Direct numerical simulation of resistive drift wave turbulence has been performed to study bifurcation phenomenon in 2D plasma turbulence. This work is to validate the recent dynamical systems approach for the L-H transition [Ball *et al*. (2002)].

$$\varepsilon \frac{\mathrm{d}P}{\mathrm{d}t} = q - \gamma P N \quad (1)$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \gamma P N - \alpha F N - \beta N^2 \quad (2)$$

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \alpha F N - \mu(P,N) F + \varphi F^{1/2} \quad (3)$$



Low dimensional modeling

- Energetics
  Diamond (1994), Sugama & Horton (1994)
- Gelerkin truncation
  Hu & Horton (1997), Kolesnikov & Krommes (2005) [Center Maniforld]

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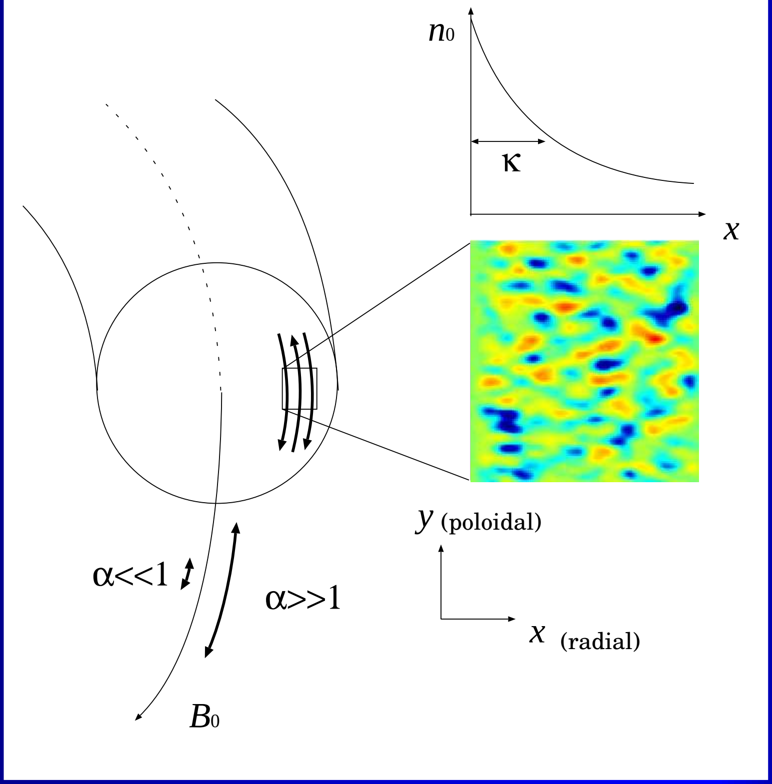
Hasegawa-Wakatani Model

HW model describes evolution of density fluctuation n and vorticity $\zeta = \nabla^2\varphi$ (φ: electrostatic potential)

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\varphi - n) - D_\zeta(-\nabla^2)^m\zeta$$

$$\frac{\partial}{\partial t}n + \{\varphi, n\} = \alpha(\varphi - n) - \kappa\frac{\partial\varphi}{\partial y} - D_n(-\nabla^2)^m n$$



$\{a, b\} = \partial a/\partial x \partial b/\partial y - \partial a/\partial y \partial b/\partial x$

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$

D_ζ and D_n are dissipation coefficients,

(Prandtl number $P_r \equiv D_\zeta/D_n$)

$\kappa \equiv -\partial/\partial x \ln n_0$

$\alpha \equiv \frac{T_e k_z^2}{\eta n_0 \omega_{ci} e^2}$: adiabaticity parameter

α

0 Hydrodynamic ... Adiabatic ∞ (Hasegawa-Mima)

Modified Hasegawa-Wakatani Model

- Resistive coupling term comes from parallel electron response $\partial j_z/\partial z = 1/\eta\partial^2(\varphi - n)/\partial z^2$ (Ohm's Law)
- Zonal components subtracted from resistive coupling term since the zonal components ($k_y = k_z = 0$) do not contribute to this term [Hammett *et al* (1993)]

Modified HW model

$$\begin{aligned}\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} =& \alpha(\tilde{\varphi} - \tilde{n}) - D_\zeta(-\nabla^2)^m\zeta \\ \frac{\partial}{\partial t}n + \{\varphi, n\} =& \alpha(\tilde{\varphi} - \tilde{n}) - \kappa\frac{\partial\varphi}{\partial y} - D_n(-\nabla^2)^m n\end{aligned}$$

where non-zonal $\tilde{\cdot}$ and zonal components $\langle\cdot\rangle$

$$\tilde{\varphi} = \varphi - \langle\varphi\rangle, \quad \tilde{n} = n - \langle n\rangle, \quad \langle f\rangle = \frac{1}{L_y}\int f\,\mathrm{d}y \quad (f = \varphi \text{ or } n)$$

Parallel wave number should be chosen to give maximum growth rate:
$\alpha = 4k^2k_y\kappa/(1+k^2)^2$ []Hasegawa & Wakatani (1984)]

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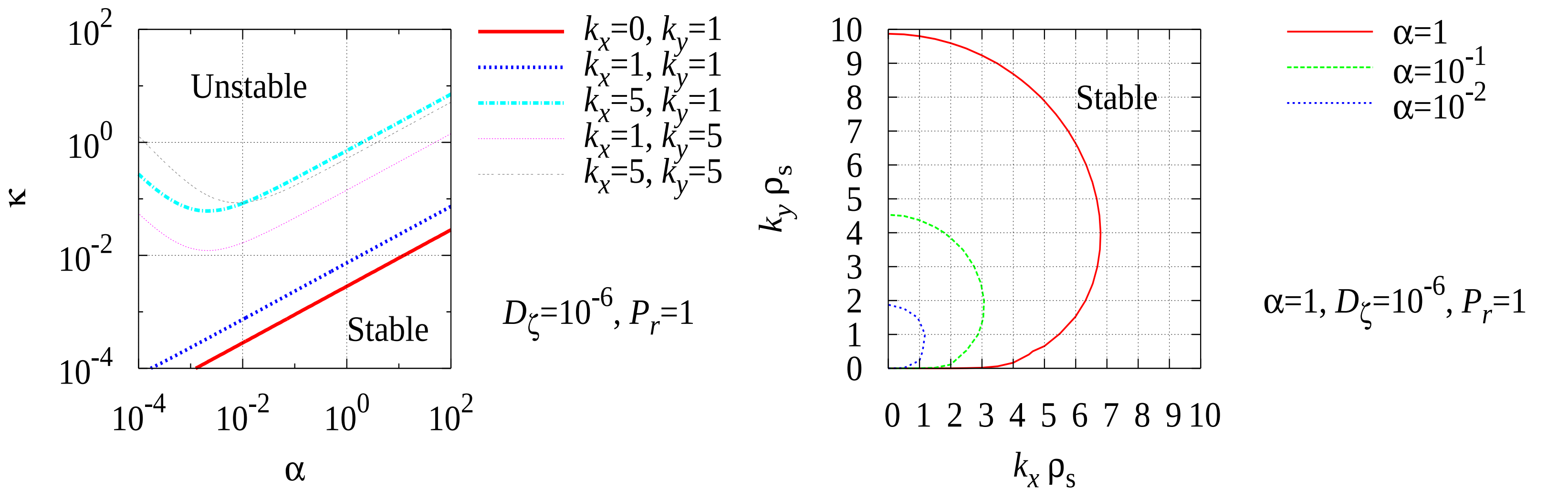
Stability Diagram Provides Indication of Transition Points

Stability threshold in α (electron adiabaticity) – κ (drive) space
There exists one linearly growing mode if $D_\zeta = D_n = 0$, which can be stabilized by finite Ds.



- Large $\kappa \longrightarrow$ destabilizing, Large $\alpha \longrightarrow$ stabilizing.
- Growth rate monotonically decreases with k_x, peaks at $k_y \sim 1$.

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Algorithm to Solve MHW Model

The code is originally developed by B. Scott, IPP.

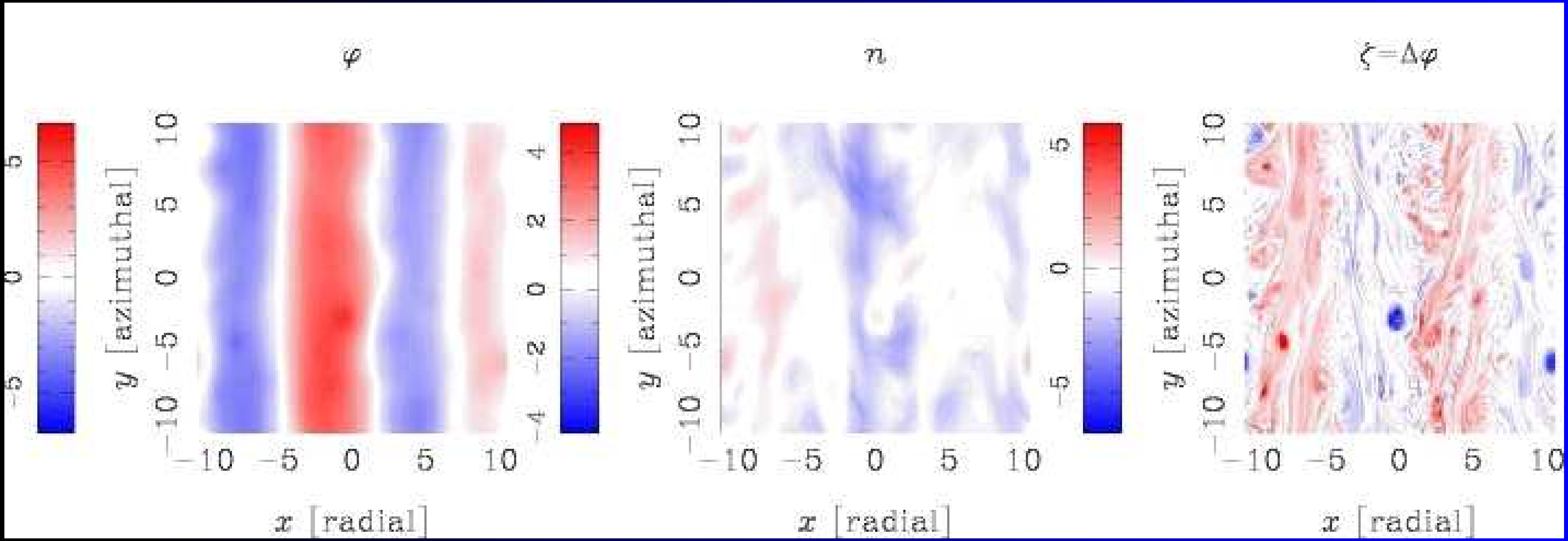
- MHW model is solved in the slab geometry
- Box size L, determined by smallest wavenumber $\Delta k = 0.3$ $[(2L)^2 = (2\pi/\Delta k)^2]$
- Periodic boundary in both x and y direction
- Time stepping algorithm is a 3rd order explicit linear multistep method (stiffly stable method)]. The method for $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = \boldsymbol{f}(t, \boldsymbol{x})$ is expressed by

$$\frac{11}{6}\boldsymbol{x}_n - 3\boldsymbol{x}_{n-1} + \frac{3}{2}\boldsymbol{x}_{n-2} - \frac{1}{3}\boldsymbol{x}_{n-3} = 3\boldsymbol{f}(t_{n-1}, \boldsymbol{x}_{n-1}) - 3\boldsymbol{f}(t_{n-2}, \boldsymbol{x}_{n-2}) + \boldsymbol{f}(t_{n-3}, \boldsymbol{x}_{n-3})$$

- Finite difference method is used for spatial discretization
- Poisson bracket term evaluated by the Arakawa's method (Arakawa (1966))
- MPI parallelized to implement on APAC-NF SGI Altix 3700 Bx2 cluster

Zonal Flow Generated in MHW Model

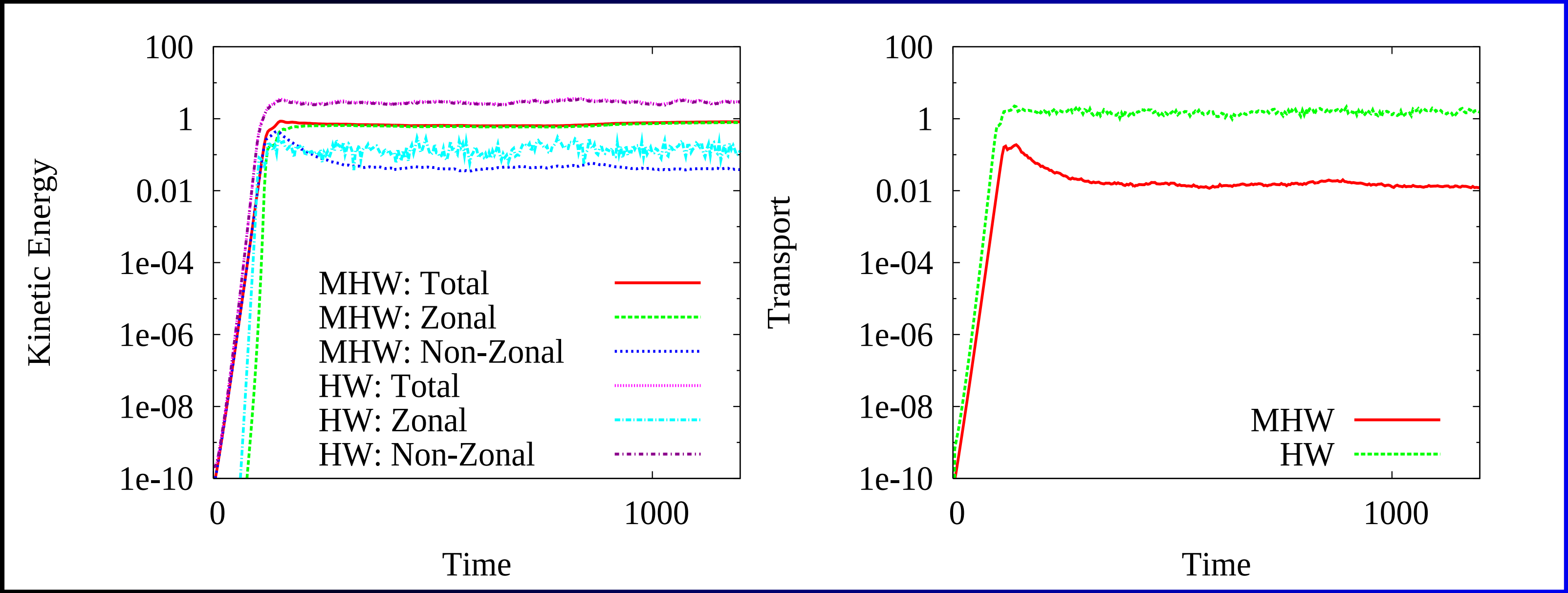
Contour plot of saturated variables for $\kappa = 1$, $\alpha = 1$, $D_\zeta = 10^{-6}$, $P_r = 1$



- Zonal flow is observed only in the MHW model (isotropic in HW model). The modification is essential for zonal flow generation.

Zonal Flow Suppresses Transport

Time evolution of kinetic energy $E = 1/2 \int |\nabla\varphi|^2 \mathrm{d}x\mathrm{d}y$ and cross-field transport $\Gamma_n = -\int \kappa(\partial\varphi/\partial y)\mathrm{d}x\mathrm{d}y$ for modified HW and original HW model



- Zonal flow components dominates kinetic energy in MHW model
- Once zonal flow is generated, transport level is significantly suppressed

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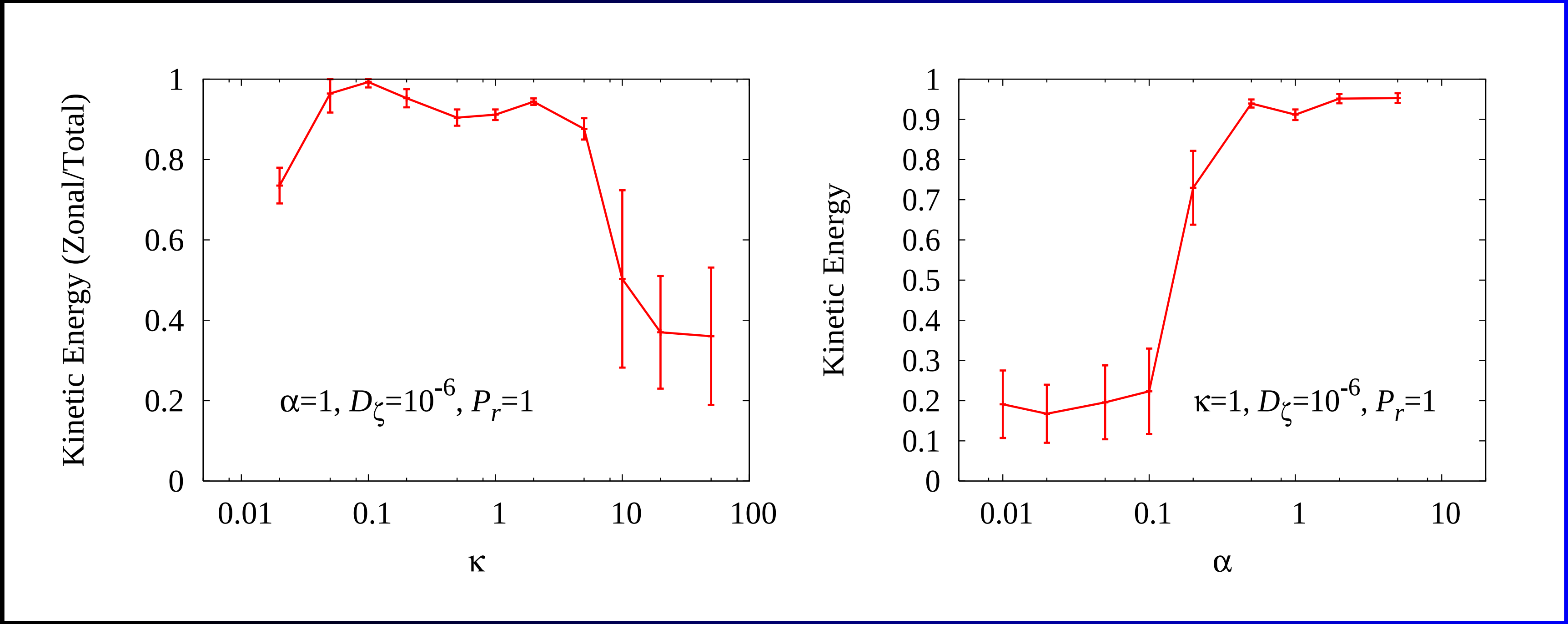
Transition to Zonal Flow Suppressed Regime Observed

Zonal kinetic energy: $F \equiv 1/2 \int \left(\frac{\partial \langle \varphi \rangle}{\partial x} \right)^2 \mathrm{d}x\mathrm{d}y$

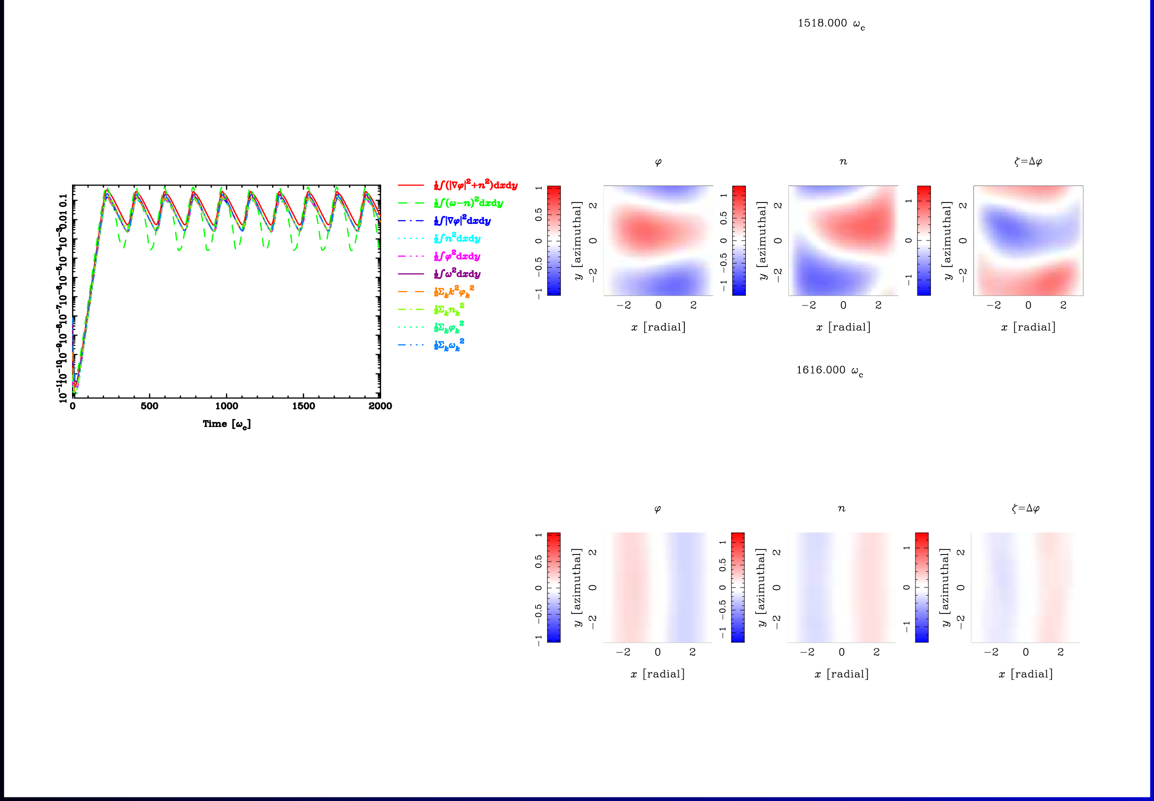


Sudden transition occurs if we move parameters to linearly more unstable direction

To Understand the Bifurcations ...

Nonlinear stability analysis for given background profile

- Zonal Flow : $\varphi_0 \propto e^{ik_x x}$
- 1st destabilized structure just beyond linear stability threshold
  Oscillatory behavior observed – Predator-Pray like



Summary and Conclusion

- We have performed simulations of modified Hasegawa-Wakatani model
- Zonal flows are generated, and the zonal flows suppresses cross-field turbulent transport in MHW model
- Transition from zonal flow dominant to zonal flow suppressed state is observed
- Bifurcations points may be understood as the destabilization of the zonal flow, or some other structure. What is it?
- Predator-Pray oscillation observed just beyond threshold [D_ζ as a control parameter]

References and Acknowledgments

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